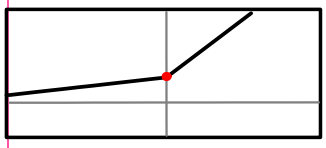
Laurent Series and z-Transform - Geometric Series Applications

A

20200718 Sat

a^n



*a

/a

Shifting a sequence

$\frac{1}{1-az}$ $|z|<a^{-1}$ — $a^n\ u(n)$

*a → $\frac{a}{1-az}$ $|z|<a^{-1}$ — $a^{n+1}\ u(n)$

/z → $\frac{az}{1-az}$ $|z|<a^{-1}$ — $a^n\ u(n-1)$

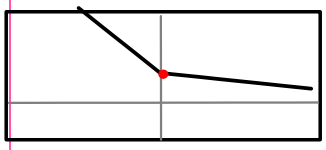
*z → $\frac{z}{1-az}$ $|z|<a^{-1}$ — $a^{n-1}\ u(n-1)$

/a → $-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^n\ u(-n-1)$

*a → $-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^{n+1}\ u(-n-1)$

/z → $-\frac{1}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^n\ u(-n)$

*z → $-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$ — $a^{n-1}\ u(-n)$

a^{-n} 

/a

*a

Shifting a sequence

$\frac{1}{1-a^{-1}z}$ $|z|<a$ — $a^{-n}\,u(n)$

/a

/z

$\frac{a^{-1}}{1-a^{-1}z}$ $|z|<a$ — $a^{-n-1}\,u(n)$

*z

$\frac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$ — $a^{-n}\,u(n-1)$

*a

$\frac{z}{1-a^{-1}z}$ $|z|<a$ — $a^{-n+1}\,u(n-1)$

$-\frac{az^{-1}}{1-az^{-1}}$ $|z|>a$ — $a^{-n}\,u(-n-1)$

/a

/z

$-\frac{z^{-1}}{1-az^{-1}}$ $|z|>a$ — $a^{-n-1}\,u(-n-1)$

*z

$-\frac{1}{1-az^{-1}}$ $|z|>a$ — $a^{-n}\,u(-n)$

*a

$-\frac{a}{1-az^{-1}}$ $|z|>a$ — $a^{-n+1}\,u(-n)$

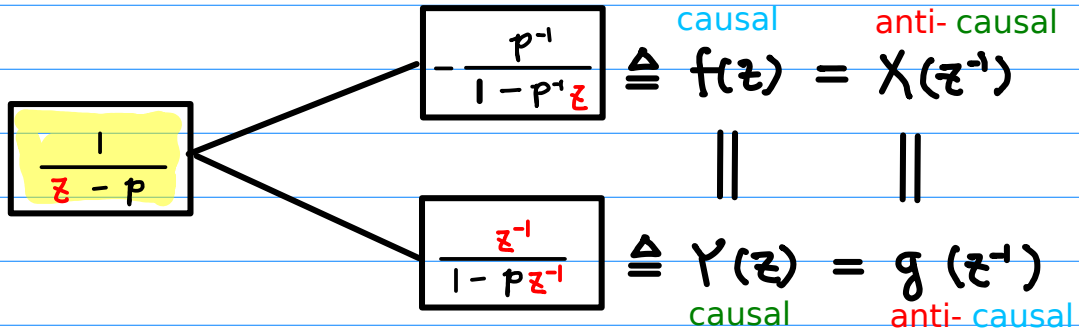
2 formulas

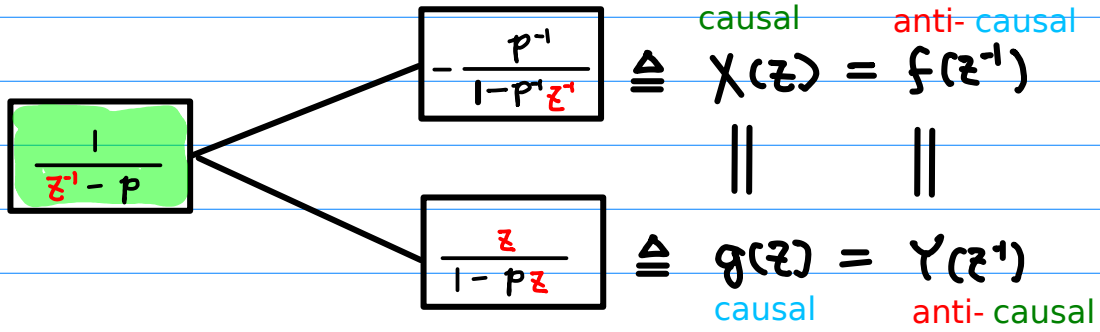
Simple Pole Form

$$\frac{1}{z - p} \qquad \frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form





Simple Pole Form	Geometric Series Form

Geometric Series (1)

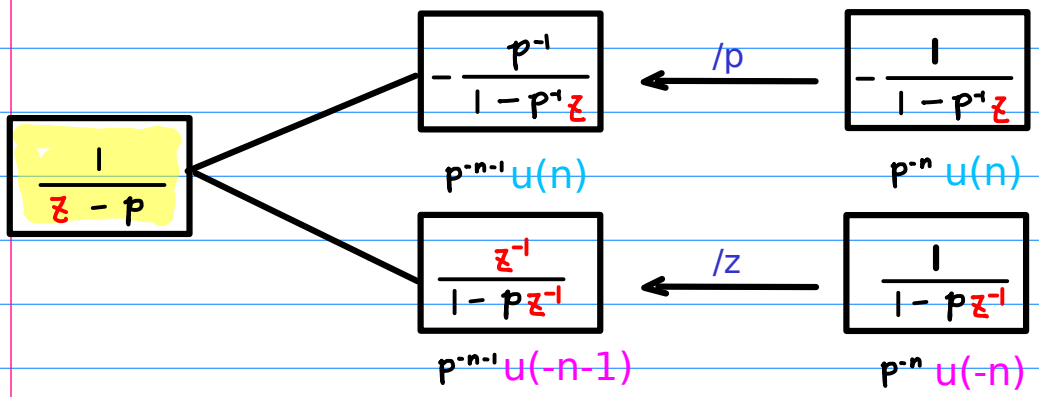
2 formulas

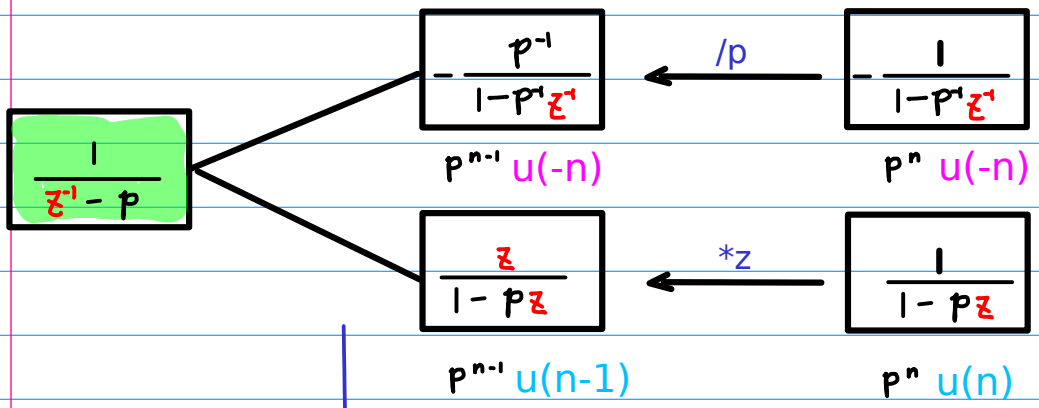
Simple Pole Form

$\frac{1}{z - p}$ $\frac{1}{z^{-1} - p}$

2 representations each

Geometric Series Form





Simple Pole Form | Geometric Series Form

Geometric Series (2)

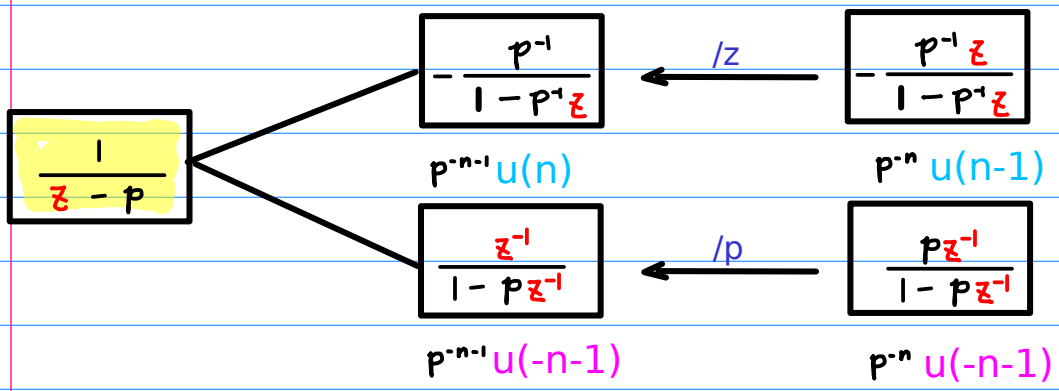
2 formulas

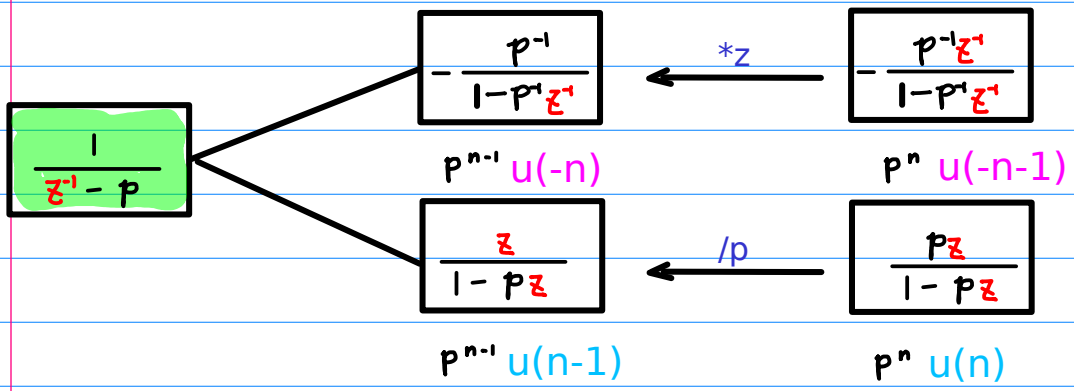
Simple Pole Form

$$\frac{1}{z - p} \qquad \frac{1}{z^{-1} - p}$$

2 representations each

Geometric Series Form





Simple Pole Form | Geometric Series Form

Geometric Series Form Combinations with a unit start term

(1) $-\dfrac{1}{1-az}$ $-a^{n}$ u(n)

(2) $-\dfrac{1}{1-az^{-1}}$ $-a^{-n}$ u(-n)

(3) $+\dfrac{1}{1-a^{-1}z^{-1}}$ a^{n} u(-n)

(4) $+\dfrac{1}{1-a^{-1}z}$ a^{-n} u(n)

(4)' $-\dfrac{1}{1-a^{-1}z}$ $-a^{-n}$ u(n)

(3)' $-\dfrac{1}{1-a^{-1}z^{-1}}$ $-a^{n}$ u(-n)

(2)' $+\dfrac{1}{1-az^{-1}}$ a^{-n} u(-n)

(1)' $+\dfrac{1}{1-az}$ a^{n} u(n)

Geometric Series Form Combinations with a common-ratio start term

(5) $+\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $a^{n}\,u(-n-1)$

(6) $+\dfrac{a^{-1}z}{1-a^{-1}z}$ $a^{-n}\,u(n-1)$

(7) $-\dfrac{az}{1-az}$ $-a^{n}\,u(n-1)$

(8) $-\dfrac{az^{-1}}{1-az^{-1}}$ $-a^{-n}u(-n-1)$

(8)' $+\dfrac{az^{-1}}{1-az^{-1}}$ $a^{-n}\,u(-n-1)$

(7)' $+\dfrac{az}{1-az}$ $a^{n}\,u(n-1)$

(6)' $-\dfrac{a^{-1}z}{1-a^{-1}z}$ $-a^{-n}\,u(n-1)$

(5)' $-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $-a^{n}\,u(-n-1)$

Geometric Series - a unit start term

Laurent Series

Geometric Series - a unit start term

z-Transform

Geometric Series - a unit start term

Laurent Series vs. z-Transform

Geometric Series - a unit start term

Laurent Series

(1) $-\frac{1}{1-az}$ $\quad |z| < a^{-1}$

$-\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$-a^n\ u(n)$	$(n \geq 0)$

(2) $-\frac{1}{1-az^{-1}}$ $\quad |z| > a$

$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

$-\left(\frac{1}{a}\right)^n\ u(-n)$	$(n < 1)$

(3) $+\frac{1}{1-a^{-1}z^{-1}}$ $\quad |z| > a^{-1}$

$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$a^n\ u(-n)$	$(n < 1)$

(4) $+\frac{1}{1-a^{-1}z}$ $\quad |z| < a$

$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

$\left(\frac{1}{a}\right)^n\ u(n)$	$(n \geq 0)$

(4)' $-\frac{1}{1-a^{-1}z}$ $\quad |z| < a$

$-\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

$-\left(\frac{1}{a}\right)^n\ u(n)$	$(n \geq 0)$

(3)' $-\frac{1}{1-a^{-1}z^{-1}}$ $\quad |z| > a^{-1}$

$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$-a^n\ u(-n)$	$(n < 1)$

(2)' $+\frac{1}{1-az^{-1}}$ $\quad |z| > a$

$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

$\left(\frac{1}{a}\right)^n\ u(-n)$	$(n < 1)$

(1)' $+\frac{1}{1-az}$ $\quad |z| < a^{-1}$

$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$a^n\ u(n)$	$(n \geq 0)$

Geometric Series - a unit start term

z-Transform (n → -n)

(1) $-\frac{1}{1-az}$ $\quad |z|<a^{-1}$

$-(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots)$

$-((\frac{1}{a})^0z^0+(\frac{1}{a})^{-1}z^{-1}+(\frac{1}{a})^{-2}z^{-2}+\cdots)$

$-a^n\ u((-n))$	$(-n\geq 0)$
$-(\frac{1}{a})^n\ u(-n)$	$(n<1)$

(2) $-\frac{1}{1-az^{-1}}$ $\quad |z|>a$

$-(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots)$

$-((\frac{1}{a})^0z^0+(\frac{1}{a})^{-1}z^{-1}+(\frac{1}{a})^{-2}z^{-2}+\cdots)$

$-(\frac{1}{a})^{-n}u(-(-n))$	$(-n<1)$
$-a^n\ u(n)$	$(n\geq 0)$

(3) $+\frac{1}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$

$(a^0z^0+a^{-1}z^{-1}+a^{-2}z^{-2}+\cdots)$

$((\frac{1}{a})^0z^0+(\frac{1}{a})^1z^{-1}+(\frac{1}{a})^2z^{-2}+\cdots)$

$a^{-n}\ u(-(-n))$	$(-n<1)$
$(\frac{1}{a})^n\ u(n)$	$(n\geq 0)$

(4) $+\frac{1}{1-a^{-1}z}$ $\quad |z|<a$

$(a^0z^0+a^1z^1+a^2z^2+\cdots)$

$((\frac{1}{a})^0z^0+(\frac{1}{a})^1z^1+(\frac{1}{a})^2z^2+\cdots)$

$(\frac{1}{a})^{-n}\ u((-n))$	$(-n\geq 0)$
$a^n\ u(-n)$	$(n<1)$

(4)' $-\frac{1}{1-a^{-1}z}$ $\quad |z|<a$

$-(a^0z^0+a^{-1}z^1+a^{-2}z^2+\cdots)$

$-((\frac{1}{a})^0z^0+(\frac{1}{a})^1z^1+(\frac{1}{a})^2z^2+\cdots)$

$-(\frac{1}{a})^{-n}\ u((-n))$	$(-n\geq 0)$
$-a^n\ u(-n)$	$(n<1)$

(3)' $-\frac{1}{1-a^{-1}z^{-1}}$ $\quad |z|>a^{-1}$

$-(a^0z^0+a^{-1}z^{-1}+a^{-2}z^{-2}+\cdots)$

$-((\frac{1}{a})^0z^0+(\frac{1}{a})^1z^{-1}+(\frac{1}{a})^2z^{-2}+\cdots)$

$-a^{-n}\ u(-(-n))$	$(-n<1)$
$-(\frac{1}{a})^n\ u(n)$	$(n\geq 0)$

(2)' $+\frac{1}{1-az^{-1}}$ $\quad |z|>a$

$(a^0z^0+a^1z^{-1}+a^2z^{-2}+\cdots)$

$((\frac{1}{a})^0z^0+(\frac{1}{a})^1z^1+(\frac{1}{a})^2z^2+\cdots)$

$(\frac{1}{a})^{-n}\ u(-(-n))$	$(-n<1)$
$a^n\ u(n)$	$(n\geq 0)$

(1)' $+\frac{1}{1-az}$ $\quad |z|<a^{-1}$

$(a^0z^0+a^1z^1+a^2z^2+\cdots)$

$((\frac{1}{a})^0z^0+(\frac{1}{a})^1z^1+(\frac{1}{a})^2z^2+\cdots)$

$a^{-n}\ u((-n))$	$(-n\geq 0)$
$(\frac{1}{a})^n\ u(-n)$	$(n<1)$

Geometric Series - a unit start term

Laurent Series vs. z-Transform ($n \to -n$)

(1) $-\frac{1}{1-az}$, $|z|<a^{-1}$

$-\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

Laurent	$-a^n \; u(n)$	$(n \geq 0)$
z-Trans	$-\left(\frac{1}{a}\right)^n \; u(-n)$	$(n<1)$

(2) $-\frac{1}{1-az^{-1}}$, $|z|>a$

$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

Laurent	$-\left(\frac{1}{a}\right)^n \; u(-n)$	$(n<1)$
z-Trans	$-a^n \; u(n)$	$(n \geq 0)$

(3) $+\frac{1}{1-a^{-1}z^{-1}}$, $|z|>a^{-1}$

$\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

Laurent	$a^n \; u(-n)$	$(n<1)$
z-Trans	$\left(\frac{1}{a}\right)^n \; u(n)$	$(n \geq 0)$

(4) $+\frac{1}{1-a^{-1}z}$, $|z|<a$

$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

Laurent	$\left(\frac{1}{a}\right)^n \; u(n)$	$(n \geq 0)$
z-Trans	$a^n \; u(-n)$	$(n<1)$

(4)' $-\frac{1}{1-a^{-1}z}$, $|z|<a$

$-\left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

Laurent	$-\left(\frac{1}{a}\right)^n \; u(n)$	$(n \geq 0)$
z-Trans	$-a^n \; u(-n)$	$(n<1)$

(3)' $-\frac{1}{1-a^{-1}z^{-1}}$, $|z|>a^{-1}$

$-\left(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

Laurent	$-a^n \; u(-n)$	$(n<1)$
z-Trans	$-\left(\frac{1}{a}\right)^n \; u(n)$	$(n \geq 0)$

(2)' $+\frac{1}{1-az^{-1}}$, $|z|>a$

$\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^{-1} + \left(\frac{1}{a}\right)^2 z^{-2} + \cdots\right)$

Laurent	$\left(\frac{1}{a}\right)^n \; u(-n)$	$(n<1)$
z-Trans	$a^n \; u(n)$	$(n \geq 0)$

(1)' $+\frac{1}{1-az}$, $|z|<a^{-1}$

$\left(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

Laurent	$a^n \; u(n)$	$(n \geq 0)$
z-Trans	$\left(\frac{1}{a}\right)^n \; u(-n)$	$(n<1)$

Geometric Series - a non-unit start term

Laurent Series

Geometric Series - a non-unit start term

z-Transform

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform

Geometric Series - a non-unit start term

Laurent Series

(5) $+\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$a^n\ u(-n-1)$ $(n<0)$

(6) $+\dfrac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$

$(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

$(\frac{1}{a})^n\ u(n-1)$ $(n\geq 1)$

(7) $-\dfrac{az}{1-az}$ $|z|<a^{-1}$

$-(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

$-a^n\ u(n-1)$ $(n\geq 1)$

(8) $-\dfrac{az^{-1}}{1-az^{-1}}$ $|z|>a$

$-(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$-((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

$-(\frac{1}{a})^n\ u(-n-1)$ $(n<0)$

(8)' $+\dfrac{az^{-1}}{1-az^{-1}}$ $|z|>a$

$(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

$(\frac{1}{a})^n u(-n-1)$ $(n<0)$

(7)' $+\dfrac{az}{1-az}$ $|z|<a^{-1}$

$(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

$a^n\ u(n-1)$ $(n\geq 1)$

(6)' $-\dfrac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$

$-(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$-((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

$-(\frac{1}{a})^n\ u(n-1)$ $(n\geq 1)$

(5)' $-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$-(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$-a^n\ u(-n-1)$ $(n<0)$

Geometric Series - a non-unit start term

z-Transform (n → -n)

(5) $+\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

$a^{-n}\ u(-(-n)-1)$	$(-n<0)$
$(\frac{1}{a})^{n}\ u(n-1)$	$(n\geq 1)$

(6) $+\dfrac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$

$(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

$(\frac{1}{a})^{-n}\ u((-n)-1)$	$(-n\geq 1)$
$a^{n}\ u(-n-1)$	$(n<0)$

(7) $-\dfrac{az}{1-az}$ $|z|<a^{-1}$

$-(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

$-((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

$-a^{-n}\ u((-n)-1)$	$(-n\geq 1)$
$-(\frac{1}{a})^{n}\ u(-n-1)$	$(n<0)$

(8) $-\dfrac{az^{-1}}{1-az^{-1}}$ $|z|>a$

$-(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$-((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

$-(\frac{1}{a})^{-n}\ u(-(-n)-1)$	$(-n<0)$
$-a^{n}\ u(n-1)$	$(n\geq 1)$

(8)' $+\dfrac{az^{-1}}{1-az^{-1}}$ $|z|>a$

$(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

$(\frac{1}{a})^{-n}\ u(-(-n)-1)$	$(-n<0)$
$a^{n}\ u(n-1)$	$(n\geq 1)$

(7)' $+\dfrac{az}{1-az}$ $|z|<a^{-1}$

$(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

$((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

$a^{-n}\ u((-n)-1)$	$(-n\geq 1)$
$(\frac{1}{a})^{n}\ u(-n-1)$	$(n<0)$

(6)' $-\dfrac{a^{-1}z}{1-a^{-1}z}$ $|z|<a$

$-(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$-((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

$-(\frac{1}{a})^{-n}\ u((-n)-1)$	$(-n\geq 1)$
$-a^{n}\ u(-n-1)$	$(n<0)$

(5)' $-\dfrac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$ $|z|>a^{-1}$

$-(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$-((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

$-a^{-n}\ u(-(-n)-1)$	$(-n<0)$
$-(\frac{1}{a})^{n}\ u(n-1)$	$(n\geq 1)$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform ($n \to -n$)

(5)

$$+\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$$(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$$

$$((\tfrac{1}{a})^{1}z^{-1} + (\tfrac{1}{a})^{2}z^{-2} + (\tfrac{1}{a})^{3}z^{-3} + \cdots)$$

Laurent	$a^{n}\ u(-n-1)$	$(n<0)$
z-Trans	$(\tfrac{1}{a})^{n}\ u(n-1)$	$(n\geq 1)$

(6)

$$+\frac{a^{-1}z}{1-a^{-1}z} \qquad |z|<a$$

$$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$$

$$((\tfrac{1}{a})^{1}z^{1} + (\tfrac{1}{a})^{2}z^{2} + (\tfrac{1}{a})^{3}z^{3} + \cdots)$$

Laurent	$(\tfrac{1}{a})^{n}\ u(n-1)$	$(n\geq 1)$
z-Trans	$a^{n}\ u(-n-1)$	$(n<0)$

(7)

$$-\frac{az}{1-az} \qquad |z|<a^{-1}$$

$$-(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$$

$$-((\tfrac{1}{a})^{1}z^{1} + (\tfrac{1}{a})^{2}z^{2} + (\tfrac{1}{a})^{3}z^{3} + \cdots)$$

Laurent	$-a^{n}\ u(n-1)$	$(n\geq 1)$
z-Trans	$-(\tfrac{1}{a})^{n}\ u(-n-1)$	$(n<0)$

(8)

$$-\frac{az^{-1}}{1-az^{-1}} \qquad |z|>a$$

$$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$$

$$-((\tfrac{1}{a})^{1}z^{-1} + (\tfrac{1}{a})^{2}z^{-2} + (\tfrac{1}{a})^{3}z^{-3} + \cdots)$$

Laurent	$-(\tfrac{1}{a})^{n}\ u(-n-1)$	$(n<0)$
z-Trans	$-a^{n}\ u(n-1)$	$(n\geq 1)$

(8)'

$$+\frac{az^{-1}}{1-az^{-1}} \qquad |z|>a$$

$$(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$$

$$((\tfrac{1}{a})^{1}z^{-1} + (\tfrac{1}{a})^{2}z^{-2} + (\tfrac{1}{a})^{3}z^{-3} + \cdots)$$

Laurent	$(\tfrac{1}{a})^{n}\ u(-n-1)$	$(n<0)$
z-Trans	$a^{n}\ u(n-1)$	$(n\geq 1)$

(7)'

$$+\frac{az}{1-az} \qquad |z|<a^{-1}$$

$$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$$

$$((\tfrac{1}{a})^{1}z^{1} + (\tfrac{1}{a})^{2}z^{2} + (\tfrac{1}{a})^{3}z^{3} + \cdots)$$

Laurent	$a^{n}\ u(n-1)$	$(n\geq 1)$
z-Trans	$(\tfrac{1}{a})^{n}\ u(-n-1)$	$(n<0)$

(6)'

$$-\frac{a^{-1}z}{1-a^{-1}z} \qquad |z|<a$$

$$-(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$$

$$-((\tfrac{1}{a})^{1}z^{1} + (\tfrac{1}{a})^{2}z^{2} + (\tfrac{1}{a})^{3}z^{3} + \cdots)$$

Laurent	$-(\tfrac{1}{a})^{n}\ u(n-1)$	$(n\geq 1)$
z-Trans	$-a^{n}\ u(-n-1)$	$(n<0)$

(5)'

$$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad |z|>a^{-1}$$

$$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$$

$$-((\tfrac{1}{a})^{1}z^{-1} + (\tfrac{1}{a})^{2}z^{-2} + (\tfrac{1}{a})^{3}z^{-3} + \cdots)$$

Laurent	$-a^{n}\ u(-n-1)$	$(n<0)$
z-Trans	$-(\tfrac{1}{a})^{n}\ u(n-1)$	$(n\geq 1)$

Complemnt ROC Pairs -
Original Geometric Series Form Combinations

(1) / (5)

unit	$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^n\ u(n)$
non-unit	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$a^n\ u(-n-1)$

(2) / (6)

$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n\ u(-n)$
$\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n\ u(n-1)$

(3) / (7)

unit	$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$a^n\ u(-n)$
non-unit	$-\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^n\ u(n-1)$

(1)

(4) / (8)

$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^n\ u(n)$
$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^n\ u(-n-1)$

(4)' / (8)'

unit	$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^n\ u(n)$
non-unit	$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^n\ u(-n-1)$

(3)' / (7)'

$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^n\ u(-n)$
$\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$a^n\ u(n-1)$

(2)' / (6)'

unit	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^n\ u(-n)$
non-unit	$-\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^n\ u(n-1)$

(1)' / (5)'

$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$a^n\ u(n)$
$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^n\ u(-n-1)$

start term

Complemnt ROC Pairs -
Shifted Geometric Series Form Combinations

(1) / (5)

$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^{n+1}$	$u(n)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	a^{n+1}	$u(-n-1)$

(2) / (6)

$-\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^{n-1}$	$u(-n)$
$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^{n-1}$	$u(n-1)$

(3) / (7)

$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	a^{n+1}	$u(-n-1)$
$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^{n+1}$	$u(n)$

(4) / (8)

$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^{n-1}$	$u(n-1)$
$\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^{n-1}$	$u(-n)$

(4)' / (8)'

$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^{n+1}$	$u(n)$
$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^{n+1}$	$u(-n-1)$

(3)' / (7)'

$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^{n-1}$	$u(-n)$
$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	a^{n-1}	$u(n-1)$

(2)' / (6)'

$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^{n+1}$	$u(-n-1)$
$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^{n+1}$	$u(n)$

(1)' / (5)'

$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	a^{n-1}	$u(n-1)$
$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^{n-1}$	$u(-n)$

Complemnt ROC Pairs - Reduced Shifted Geometric Series Form Combinations

$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$-a^{n+1}\ u(n)$	$-\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\left(\frac{1}{a}\right)^{n-1}\ u(-n)$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$a^{n+1}\ u(-n-1)$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\left(\frac{1}{a}\right)^{n-1}\ u(n-1)$

$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\left(\frac{1}{a}\right)^{n+1}\ u(n)$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-a^{n-1}\ u(-n)$
$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\left(\frac{1}{a}\right)^{n+1}\ u(-n-1)$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	$a^{n-1}\ u(n-1)$

$2z$	$2z^{-1}$	$2^{-1}z^{-1}$	$2^{-1}z$
$\lvert z\rvert < 0.5$	$\lvert z\rvert > 2$	$\lvert z\rvert > 0.5$	$\lvert z\rvert < 2$

$$-\frac{2}{1-2z} \xleftrightarrow{z^{-1}} -\frac{2}{1-2z^{-1}}$$

$$\cdot\frac{(2z)^{-1}}{(2z)^{-1}} \cdot\frac{(2z)}{(2z)} \qquad \cdot\frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}} \cdot\frac{(2z^{-1})}{(2z^{-1})}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} \xleftrightarrow{z^{-1}} +\frac{z}{1-0.5z}$$

(1) / (5)

scale(a)

$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

(2) / (6)

scale(a)

$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
Comp.ROC	$\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$

(3) / (7)

scale(1/z)

$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
Comp.ROC	$-\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

(4) / (8)

scale(z)

$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

(4)' / (8)'

scale(1/a)

$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$

(3)' / (7)'

scale(1/a)

$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
Comp.ROC	$\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$

(2)' / (6)'

scale(1/z)

$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

(1)' / (5)'

scale(z)

$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
Comp.ROC	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

(1) / (5)

scale(a)

$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

Comp.ROC

(2) / (6)

scale(a)

$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
$\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$

Comp.ROC

(3) / (7)

scale(1/z)

$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$-\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

Comp.ROC

(4) / (8)

scale(z)

$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

Comp.ROC

(4)' / (8)'

scale(1/a)

$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$

Comp.ROC

(3)' / (7)'

scale(1/a)

$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$

Comp.ROC

(2)' / (6)'

scale(1/z)

$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$-\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

Comp.ROC

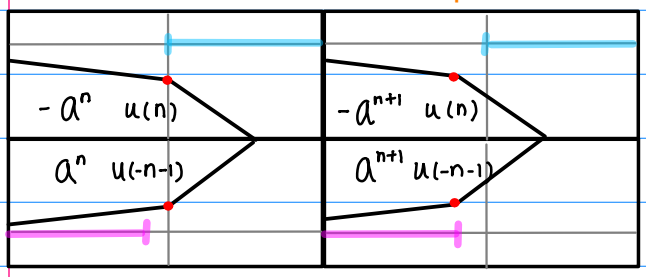
(1)' / (5)'

scale(z)

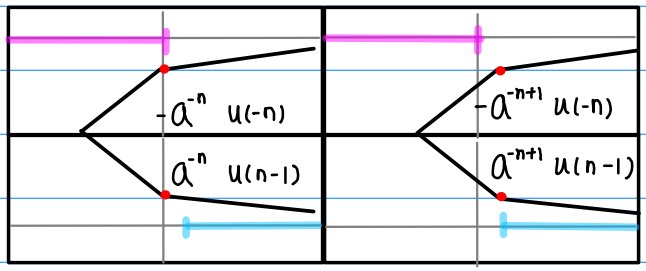
$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

SHL.Seq

-Comp.Rng

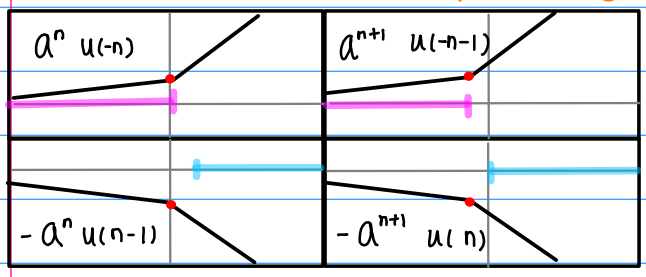


SHR.Seq

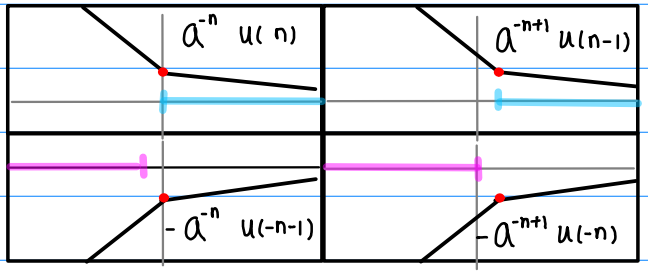


SHL.Seq, SHL.Rng

-Comp.Rng

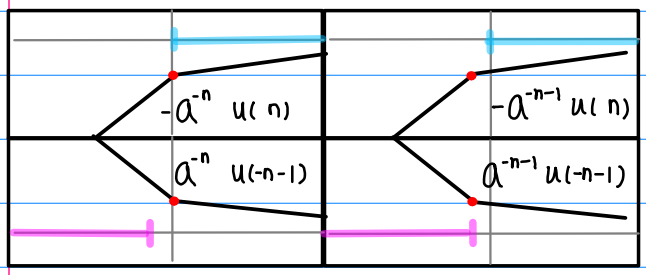


SHR.Seq, SHR.Rng

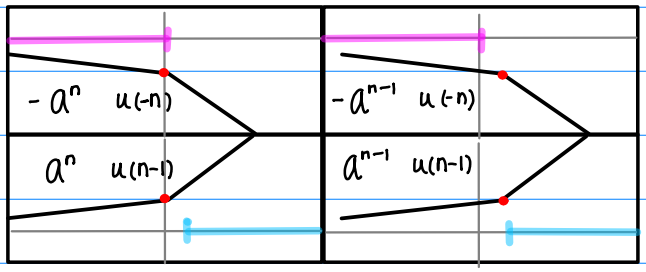


SHL.Seq

-Comp.Rng

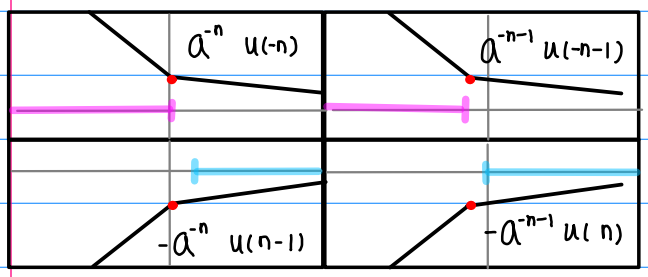


SHR.Seq

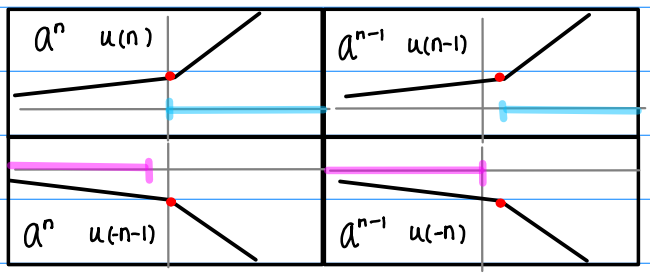


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng



Left Shifted Sequence

Right Shifted Sequence

(1) / (5)

scale(a)

(2) / (6)

scale(a)

	$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$-\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
Comp.ROC	$\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$

(3) / (7)

scale(1/z)

(4) / (8)

scale(z)

	$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
Comp.ROC	$-\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$-\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

(4)' / (8)'

scale(1/a)

(3)' / (7)'

scale(1/a)

	$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
Comp.ROC	$\frac{az^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{az}{1-az}$	$\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$

(2)' / (6)'

scale(1/z)

(1)' / (5)'

scale(z)

	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$\frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
Comp.ROC	$-\frac{a^{-1}z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$-\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

SHL.Seq

$-a^{n}\ u(n)$ $-(a^{0}, a^{1}, a^{2}, \cdots)$	$-a^{n+1}\ u(n)$ $-(a^{1}, a^{2}, a^{3}, \cdots)$
$a^{n}\ u(-n-1)$ $(\cdots, \frac{1}{a^{3}}, \frac{1}{a^{2}}, \frac{1}{a^{1}})$	$a^{n+1}\ u(-n-1)$ $(\cdots, \frac{1}{a^{2}}, \frac{1}{a^{1}}, \frac{1}{a^{0}})$

-Comp.Rng

SHR.Seq

$-(\frac{1}{a})^{n}\ u(-n)$ $-(\cdots, a^{3}, a^{2}, a^{1}, a^{0})$	$-(\frac{1}{a})^{n-1}\ u(-n)$ $-(\cdots, a^{3}, a^{2}, a^{1})$
$(\frac{1}{a})^{n}\ u(n-1)$ $(\frac{1}{a^{1}}, \frac{1}{a^{2}}, \frac{1}{a^{3}}, \cdots)$	$(\frac{1}{a})^{n-1}\ u(n-1)$ $(\frac{1}{a^{0}}, \frac{1}{a^{1}}, \frac{1}{a^{2}}, \cdots)$

SHL.Seq, SHL.Rng

$a^{n}\ u(-n)$ $(\cdots, \frac{1}{a^{2}}, \frac{1}{a^{1}}, \frac{1}{a^{0}})$	$a^{n+1}\ u(-n-1)$ $(\cdots, \frac{1}{a^{2}}, \frac{1}{a^{1}}, \frac{1}{a^{0}})$
$-a^{n}\ u(n-1)$ $-(a^{1}, a^{2}, a^{3}, \cdots)$	$-a^{n+1}\ u(n)$ $-(a^{1}, a^{2}, a^{3}, \cdots)$

-Comp.Rng

SHR.Seq, SHR.Rng

$(\frac{1}{a})^{n}\ u(n)$ $(\frac{1}{a^{0}}, \frac{1}{a^{1}}, \frac{1}{a^{2}}, \cdots)$	$(\frac{1}{a})^{n-1}\ u(n-1)$ $(\frac{1}{a^{0}}, \frac{1}{a^{1}}, \frac{1}{a^{2}}, \cdots)$
$-(\frac{1}{a})^{n}\ u(-n-1)$ $(\cdots, a^{3}, a^{2}, a^{1})$	$-(\frac{1}{a})^{n-1}\ u(-n)$ $(\cdots, a^{3}, a^{2}, a^{1})$

SHL.Seq

$-(\frac{1}{a})^{n}\ u(n)$ $-(\frac{1}{a^{0}}, \frac{1}{a^{1}}, \frac{1}{a^{2}}, \cdots)$	$-(\frac{1}{a})^{n+1}\ u(n)$ $-(\frac{1}{a^{1}}, \frac{1}{a^{2}}, \frac{1}{a^{3}}, \cdots)$
$(\frac{1}{a})^{n}\ u(-n-1)$ $(\cdots, a^{3}, a^{2}, a^{1})$	$(\frac{1}{a})^{n+1}\ u(-n-1)$ $(\cdots, a^{2}, a^{1}, a^{0})$

-Comp.Rng

SHR.Seq

$-a^{n}\ u(-n)$ $-(\cdots, \frac{1}{a^{2}}, \frac{1}{a^{1}}, \frac{1}{a^{0}})$	$-a^{n-1}\ u(-n)$ $-(\cdots, \frac{1}{a^{2}}, \frac{1}{a^{3}}, \frac{1}{a^{1}})$
$a^{n}\ u(n-1)$ $(a^{1}, a^{2}, a^{3}, \cdots)$	$a^{n-1}\ u(n-1)$ $(a^{0}, a^{1}, a^{2}, \cdots)$

SHL.Seq, SHL.Rng

$(\frac{1}{a})^{n}\ u(-n)$ $(\cdots, a^{2}, a^{1}, a^{0})$	$(\frac{1}{a})^{n+1}\ u(-n-1)$ $(\cdots, a^{2}, a^{1}, a^{0})$
$-(\frac{1}{a})^{n}\ u(n-1)$ $-(\frac{1}{a^{1}}, \frac{1}{a^{2}}, \frac{1}{a^{3}}, \cdots)$	$-(\frac{1}{a})^{n+1}\ u(n)$ $-(\frac{1}{a^{1}}, \frac{1}{a^{2}}, \frac{1}{a^{3}}, \cdots)$

-Comp.Rng

SHR.Seq, SHR.Rng

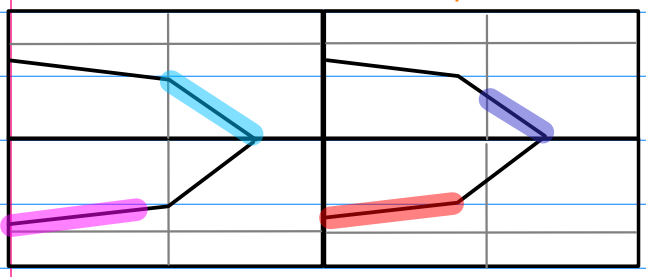
$a^{n}\ u(n)$ $(a^{0}, a^{1}, a^{2}, \cdots)$	$a^{n-1}\ u(n-1)$ $(a^{0}, a^{1}, a^{2}, \cdots)$
$-a^{n}\ u(-n-1)$ $-(\cdots, \frac{1}{a^{3}}, \frac{1}{a^{2}}, \frac{1}{a^{1}})$	$-a^{n-1}\ u(-n)$ $-(\cdots, \frac{1}{a^{3}}, \frac{1}{a^{2}}, \frac{1}{a^{1}})$

Left Shifted Sequence

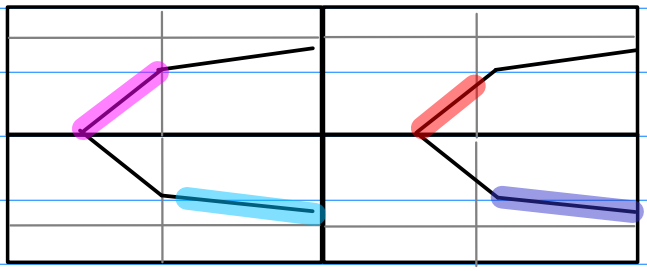
Right Shifted Sequence

-Comp.Rng

SHL.Seq

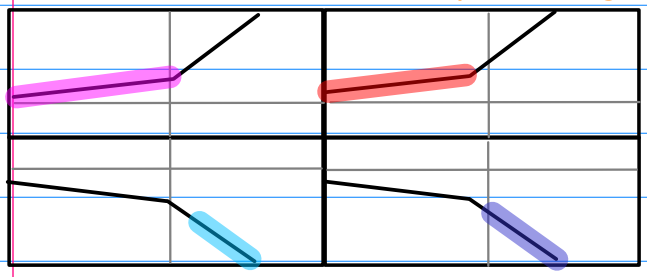


SHR.Seq

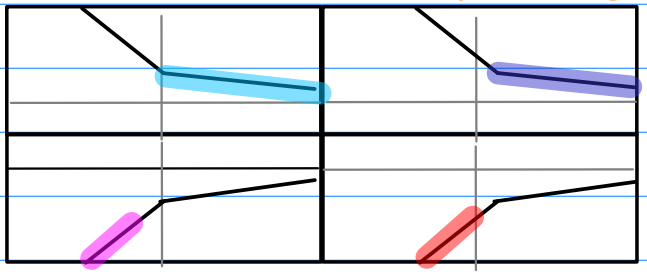


-Comp.Rng

SHL.Seq, SHL.Rng

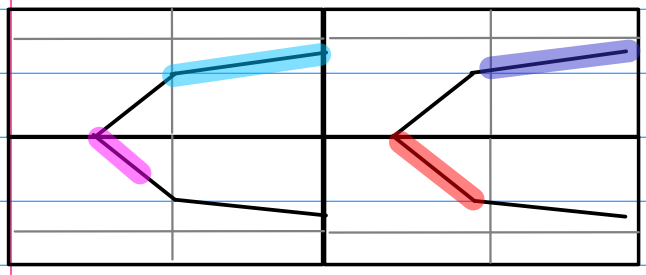


SHR.Seq, SHR.Rng

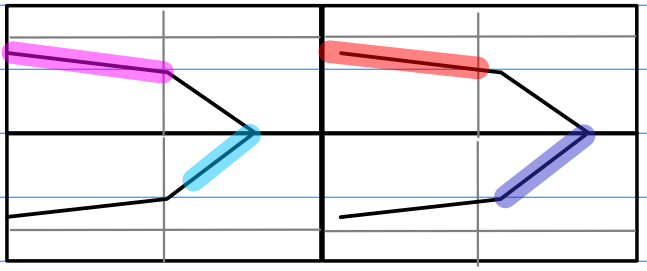


-Comp.Rng

SHL.Seq

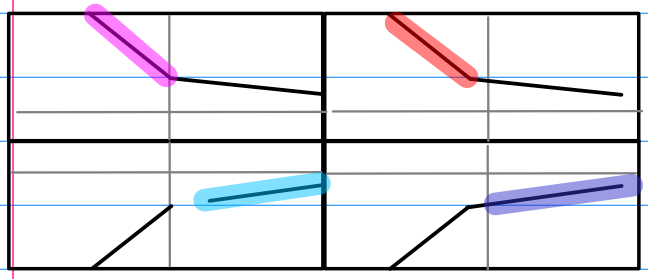


SHR.Seq

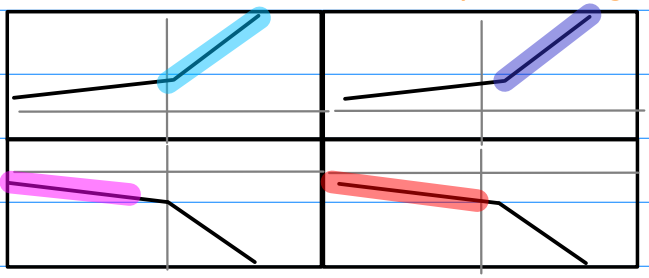


-Comp.Rng

SHL.Seq, SHL.Rng



SHR.Seq, SHR.Rng

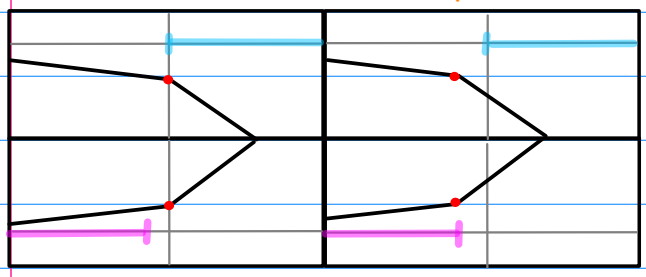


Left Shifted Sequence

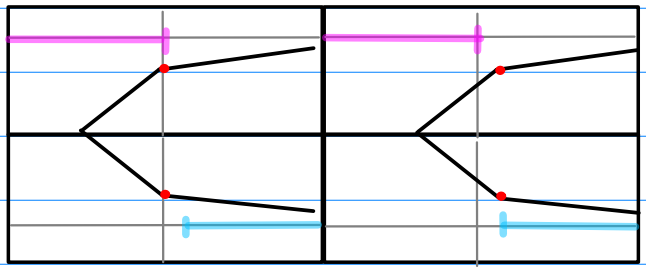
Right Shifted Sequence

SHL.Seq

-Comp.Rng

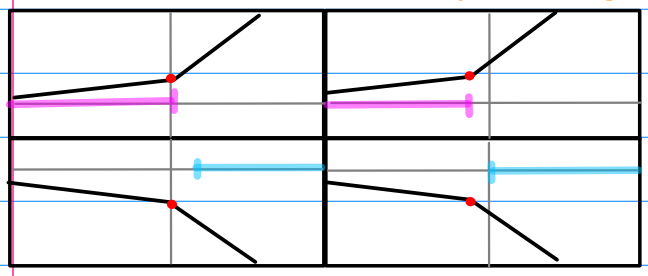


SHR.Seq

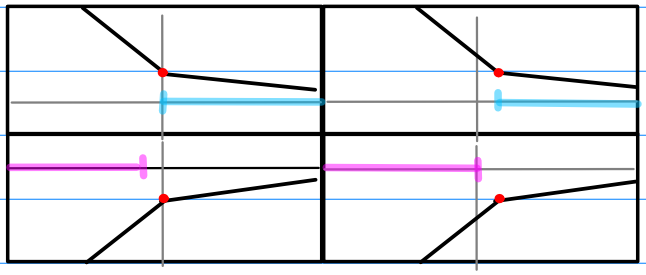


SHL.Seq, SHL.Rng

-Comp.Rng

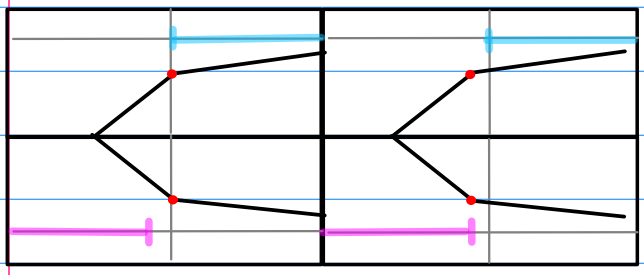


SHR.Seq, SHR.Rng

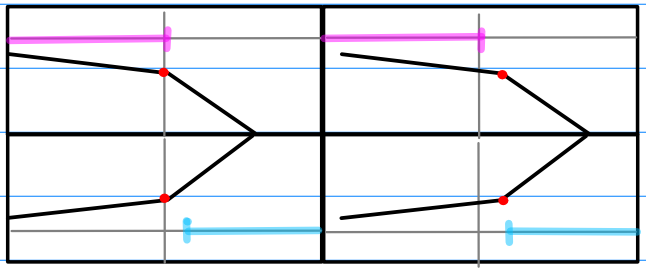


SHL.Seq

-Comp.Rng

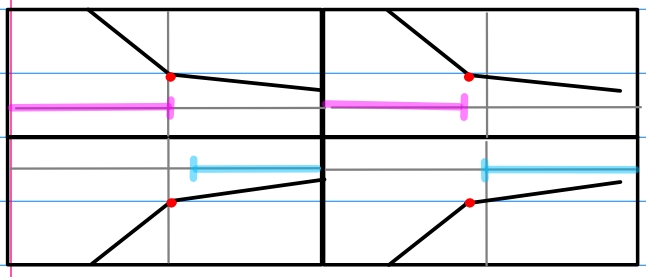


SHR.Seq

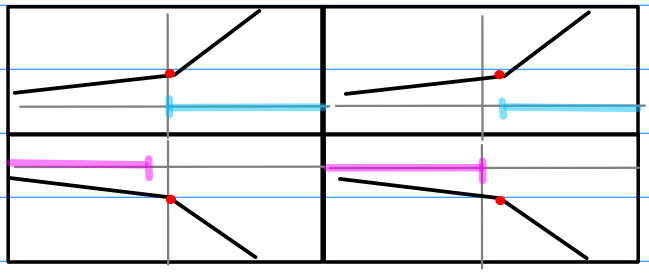


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng

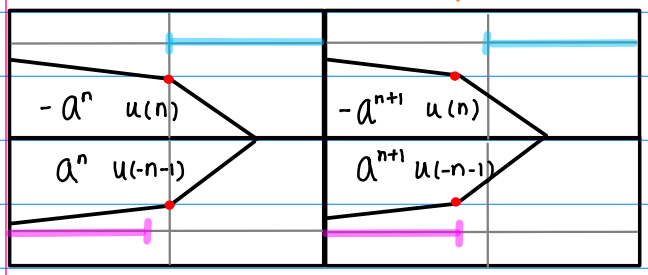


Left Shifted Sequence

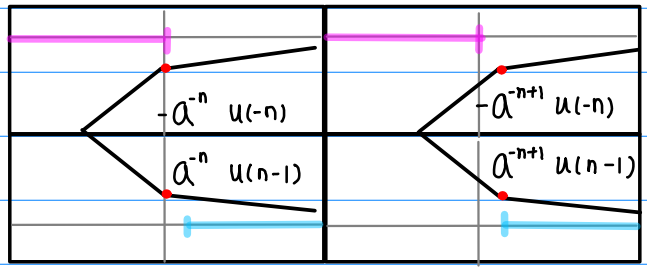
Right Shifted Sequence

SHL.Seq

-Comp.Rng

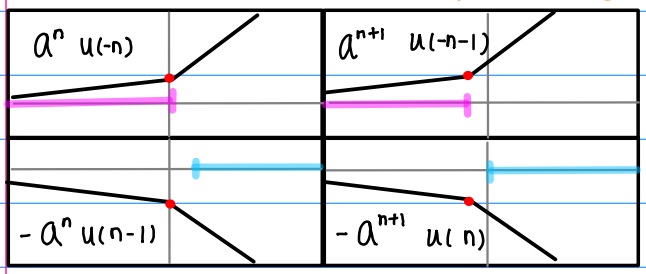


SHR.Seq

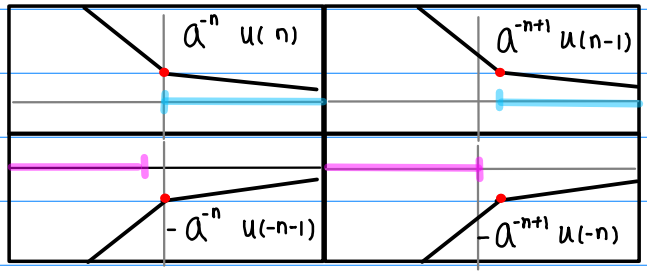


SHL.Seq, SHL.Rng

-Comp.Rng

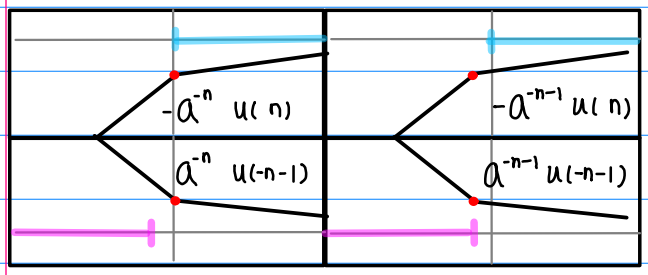


SHR.Seq, SHR.Rng

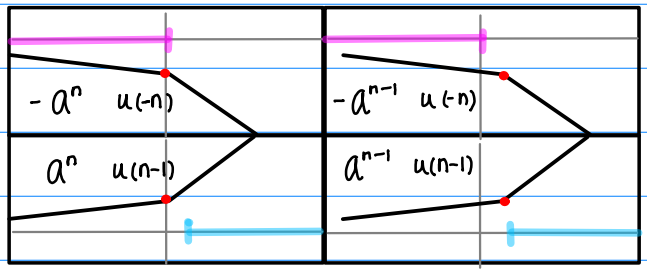


SHL.Seq

-Comp.Rng

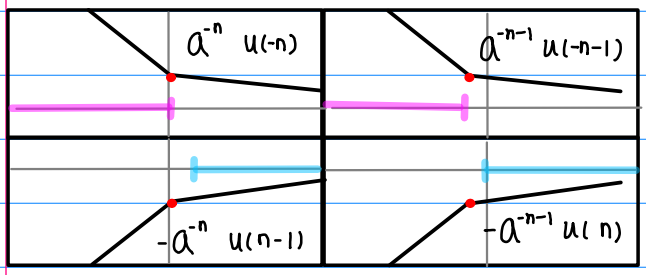


SHR.Seq

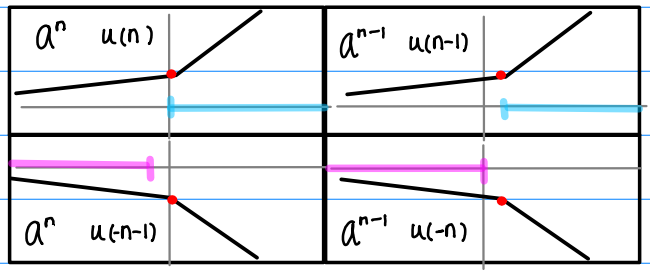


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng



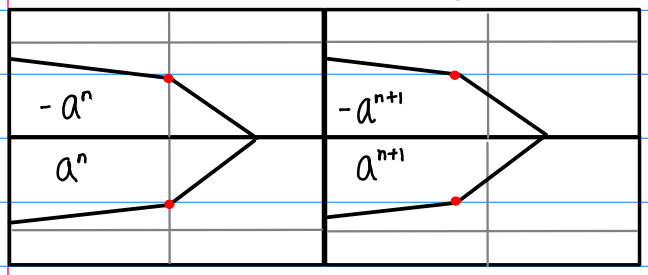
Left Shifted Sequence

Right Shifted Sequence

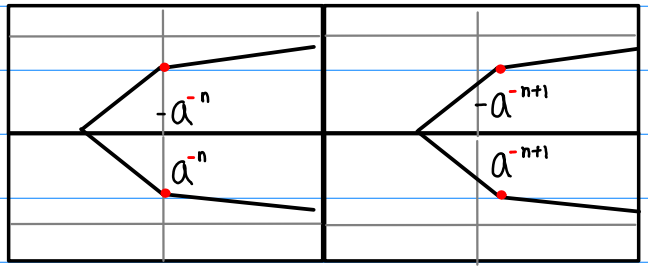
a Sequence Function

SHL.Seq

-Comp.Rng

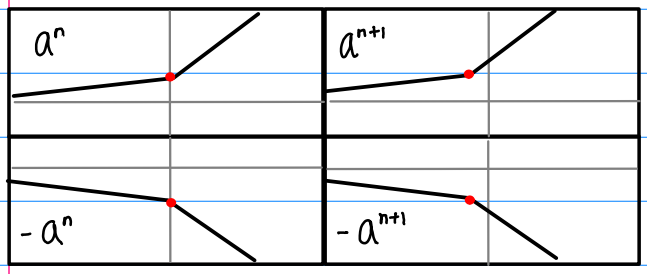


SHR.Seq

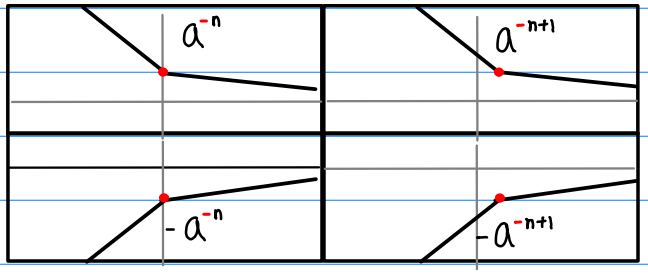


SHL.Seq, SHL.Rng

-Comp.Rng

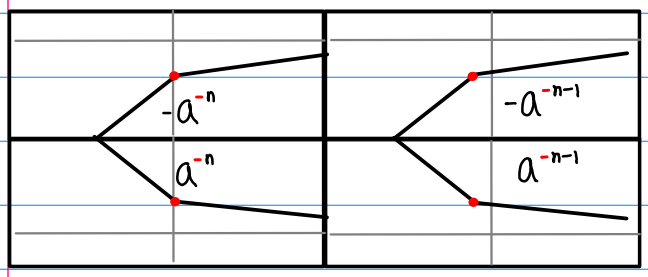


SHR.Seq, SHR.Rng

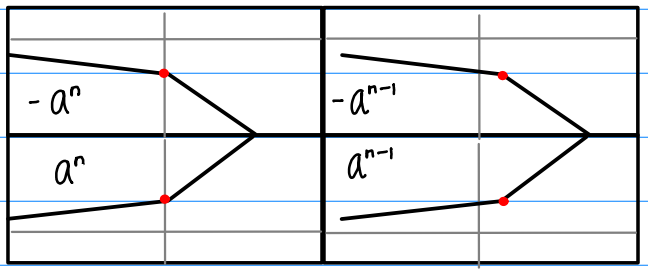


SHL.Seq

-Comp.Rng

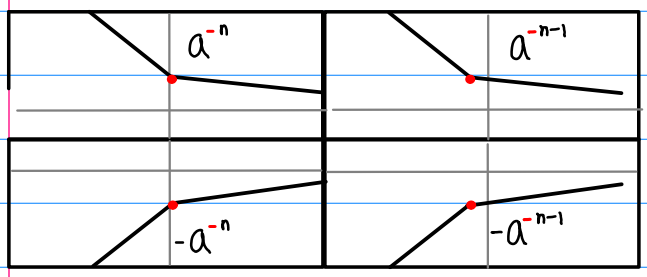


SHR.Seq

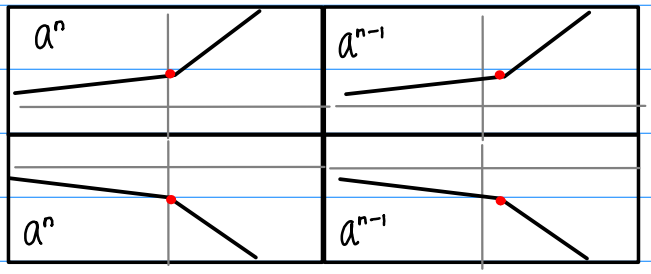


SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng



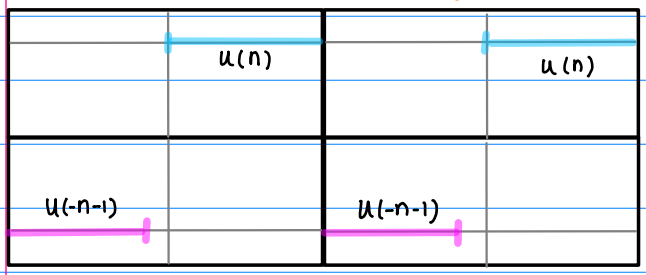
Left Shifted Sequence

Right Shifted Sequence

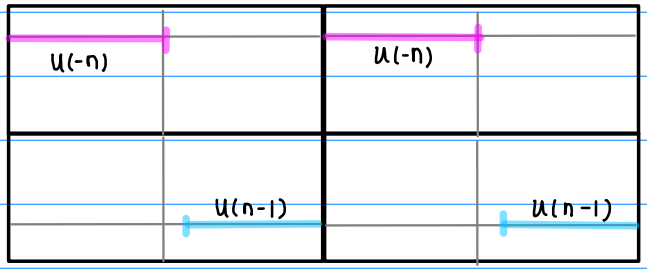
Range of a Sequence

SHL.Seq

-Comp.Rng

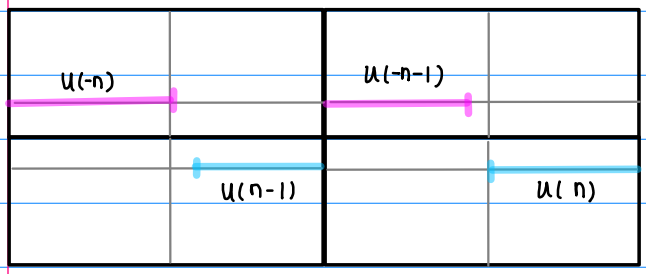


SHR.Seq

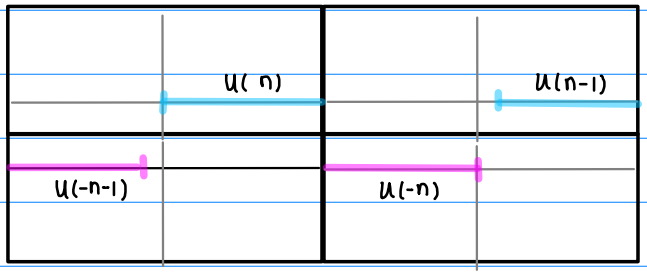


SHL.Seq, SHL.Rng

-Comp.Rng

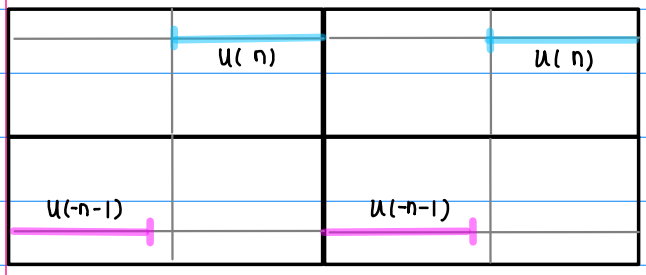


SHR.Seq, SHR.Rng

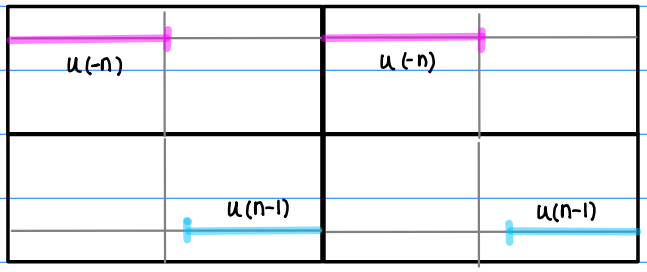


SHL.Seq

-Comp.Rng

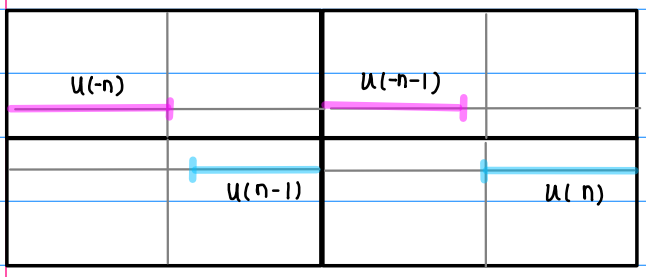


SHR.Seq



SHL.Seq, SHL.Rng

-Comp.Rng



SHR.Seq, SHR.Rng