Logic Background (1B)

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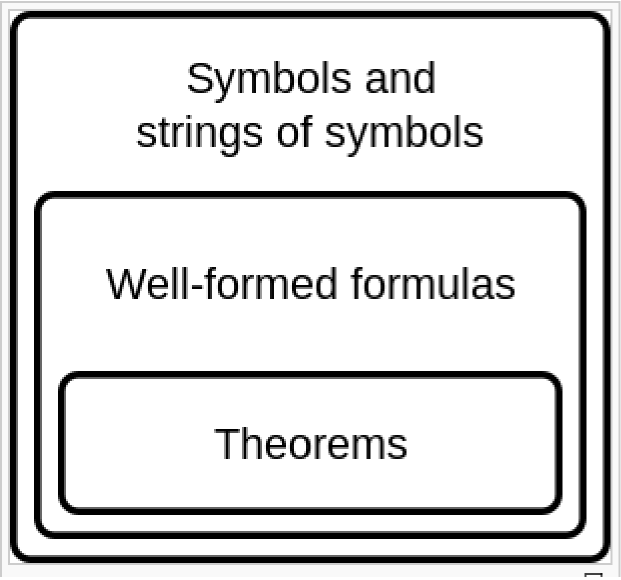
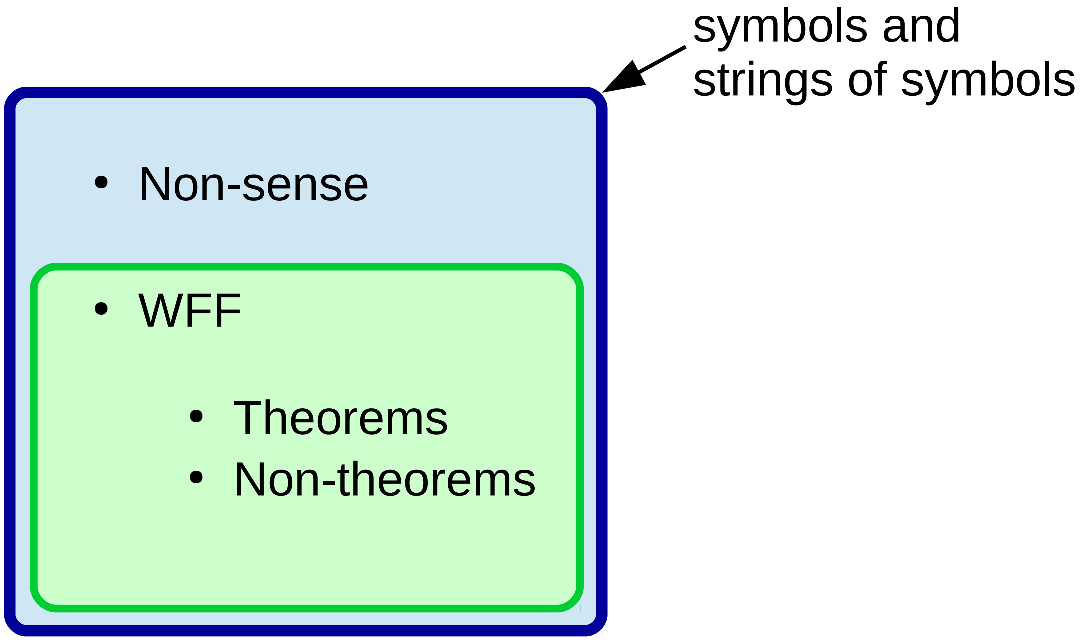
Symbols and Formal Language



This diagram shows the syntactic entities that may be constructed from formal languages. The symbols and strings of symbols may be broadly divided into nonsense and well-formed formulas. A formal language can be thought of as identical to the set of its well-formed formulas. The set of well-formed formulas may be broadly divided into theorems and non-theorems.



http://en.wikipedia.org/wiki/

Syntactic entities from formal languages

the **syntactic entities**
constructed from **formal languages**.

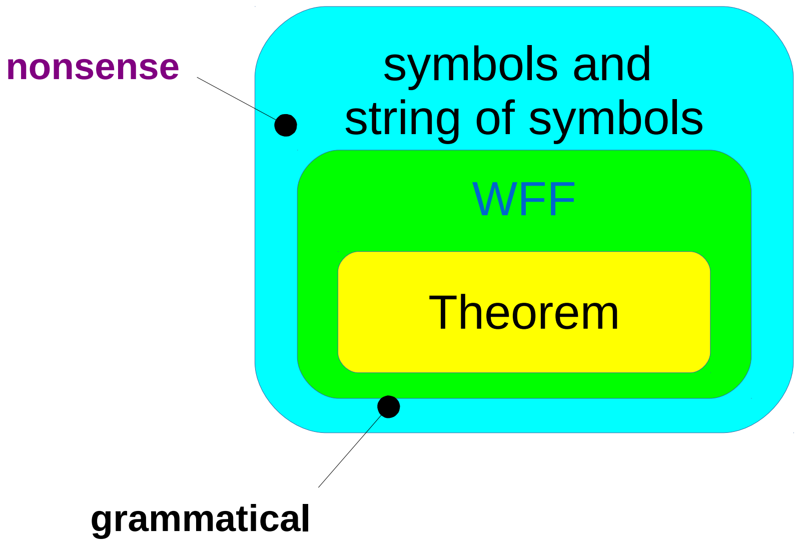
The **symbols** and **strings of symbols**

- **nonsense**
- **well-formed formulas**.

A **formal language**
the **set of its well-formed formulas**.

The set of well-formed formulas

- **theorems**
- **non-theorems**.



http://en.wikipedia.org/wiki/

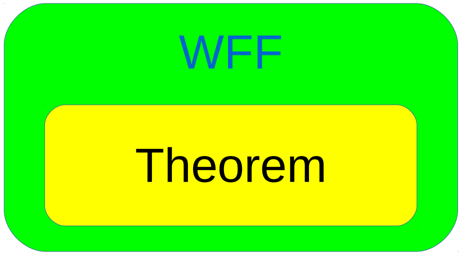
Well-formedness

Well-formedness
the quality of a clause, word, or other linguistic element that conforms to the **grammar** of the **language** of which it is a part.

Well-formed words or **phrases** are **grammatical**, meaning they obey all relevant rules of **grammar**.

a form that violates some **grammar rule** is **ill-formed** and does not constitute part of the language.

WFF is a **word**
a finite sequence of symbols from a given alphabet which is part of a **formal language**.



grammatical

http://en.wikipedia.org/wiki/

Theorem

In mathematics, a **theorem** is
a statement that has been proven on

- other **theorems**
  previously established statements
- **axioms**
  generally accepted statements

A **theorem** is a
logical consequence of the **axioms**.

Theorem

proofs

sequences of **formulas**
with certain **properties**

http://en.wikipedia.org/wiki/

Proof

The **proof** of a **mathematical theorem** is
a **logical argument**
for the **theorem statement** given
in accord with the **rules**
of a **deductive system**.

the **proof** of a **theorem**
is often interpreted as
justification of the **truth**
of the **theorem** statement.

In **formal logic**, **proofs** can be represented
by sequences of **formulas** with certain **properties**,
and the **final formula** in the sequence is what is proven.

Theorem

proofs

sequences of **formulas**
with certain **properties**

http://en.wikipedia.org/wiki/

Formula

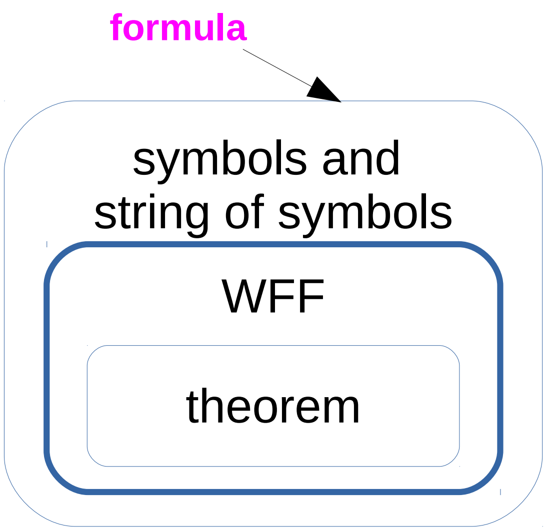
A **formula** is a **syntactic formal object** that can be informally given a **semantic meaning**.

a **formula** is **a string of symbols φ** for which it makes sense to ask "**is φ true?**", once any free variables in φ have been **instantiated**.

A key use of **formula** is

- **propositional logic**
- **predicate logics** such as **first-order logic**.

A **formal language can be considered to be identical to the set** containing all and only its **formula.**



formal logic

grammatical

http://en.wikipedia.org/wiki/

Predicate

predicate (*plural* **predicates**)

1. (*grammar*) The part of the sentence (or clause) which states something about the subject or the object of the sentence. [quotations ▼]

   *In "The dog barked very loudly", the subject is "the dog" and the **predicate** is "barked very loudly".*

2. (*logic*) A term of a statement, where the statement may be true or false depending on whether the thing referred to by the values of the statement's variables has the property signified by that (predicative) term. [quotations ▼]

   *A nullary **predicate** is a proposition. Also, an instance of a **predicate** whose terms are all constant — e.g., P(2,3) — acts as a proposition.*

   *A **predicate** can be thought of as either a relation (between elements of the domain of discourse) or as a truth-valued function (of said elements).*

   *A **predicate** is either valid, satisfiable, or unsatisfiable.*

   *There are two ways of binding a **predicate**'s variables: one is to assign constant values to those variables, the other is to quantify over those variables (using universal or existential quantifiers). If all of a **predicate**'s variables are bound, the resulting formula is a proposition.*

3. (*computing*) An operator or function that returns either true or false.

http://en.wikipedia.org/wiki/

Predicate in mathematics

In mathematics, a **predicate** is commonly understood to be a Boolean-valued function $P: X \to \{\text{true}, \text{false}\}$, called the predicate on X. However, predicates have many different uses and interpretations in mathematics and logic, and their precise definition, meaning and use will vary from theory to theory. So, for example, when a theory defines the concept of a relation, then a predicate is simply the characteristic function or the indicator function of a relation. However, not all theories have relations, or are founded on set theory, and so one must be careful with the proper definition and semantic interpretation of a predicate.

http://en.wikipedia.org/wiki/

First-order Logic

First-order logic (**predicate logic**, **first-order predicate calculus**)

a collection of **formal systems** used in mathematics,
philosophy, linguistics, and computer science.

First-order logic uses **quantified variables** over non-logical objects and
allows the use of **sentences** that contain **variables**

unlike propositions such as Socrates is a man one
can have expressions in the form
"there exists X such that X is Socrates and X is a man"
and there exists is a **quantifier** while X is a **variable**.

This distinguishes it from propositional logic,
which does not use **quantifiers** or **relations**;
propositional logic is the foundation of first-order logic.

http://en.wikipedia.org/wiki/

Propositional logic

Propositional logic

- consists of a set of atomic propositional symbols
- e.g. Socrates, Father, etc
- often referred to by letters p, q, r etc.
- these letters are not variables
- propositional logic has no means of binding variables.
- these symbols are joined together by logical operators (or connectives) to form sentences.
- can only talk about specifics
- e.g. "Socrates is a man"

https://www.quora.com/What-is-the-difference-between-predicate-logic-first-order-logic-second-order-logic-and-higher-order-logic

1st-order logic

First-order Predicate Logic

- is an extension of propositional logic
- allows **quantification** over variables.
- can also talk more generally
- e.g. "all men are mortal"
- **variables** to range over atomic symbols in the domain.
- doesn't allow variables to be bound to predicate symbols

https://www.quora.com/What-is-the-difference-between-predicate-logic-first-order-logic-second-order-logic-and-higher-order-logic

2nd-order logic

A **second order logic** (such as second order predicate logic)

- allow variables to be bound to predicate symbols
- can write sentences such as: ∀p.p(Socrates).

A **higher order logic** allows

- predicates to accept arguments which are themselves predicates.
- **Second order logic** cannot be reduced to **first-order logic**.

https://www.quora.com/What-is-the-difference-between-predicate-logic-first-order-logic-second-order-logic-and-higher-order-logic

First-Order Logic (1)

The definition of a **formula** comes in several parts.

1. the set of **terms** is defined recursively.
terms, informally, are **expressions**
that represent **objects** from the domain of discourse.

Any **variable** is a **term**.
Any **constant** **symbol** from the signature is a **term**
an expression of the form $\mathbf{f(t_1,...,t_n)}$,
where **f** is an **n-ary function** symbol, and $\mathbf{t_1},...,\mathbf{t_n}$ are terms, is again a **term**.

2. the **atomic formulas**.

If $\mathbf{t_1}$ and $\mathbf{t_2}$ are **terms** then $\mathbf{t_1} = \mathbf{t_2}$ is an **atomic formula**
If **R** is an **n-ary relation** symbol,
and $\mathbf{t_1},...,\mathbf{t_n}$ are **terms**, then $\mathbf{R(t_1,...,t_n)}$ is an **atomic formula**

formula

1. the set of **terms**
   - a **variable**
   - a **constant**
   - $\mathbf{f(t_1,...,t_n)}$,

2. the **atomic formulas**.
   - $\mathbf{t_1} = \mathbf{t_2}$
   - $\mathbf{R(t_1,...,t_n)}$

3. the set of **formulas**
   - $\neg \phi$
   - $(\phi \land \psi), (\phi \lor \psi)$
   - $\exists x \, \phi$
   - $\forall x \, \phi$

https://en.wikipedia.org/wiki/Well-formed_formula

First-Order Logic (2)

3. the set of **formulas** is defined to be the <u>smallest</u> <u>set</u> containing the set of **atomic formulas** such that the following holds:

- ¬ ϕ is a **formula** when ϕ is a **formula**
- (ϕ ∧ ψ) and (ϕ ∨ ψ) are **formulas** when ϕ and ψ are **formulas**;
- ∃ x ϕ is a **formula** when x is a **variable** and ϕ is a **formula**;
- ∀ x ϕ is a **formula** when x is a **variable** and ϕ is a **formula**
  (alternatively, ∀x ϕ could be defined as an abbreviation for ¬∃x ¬ϕ).

If a formula has no occurrences of ∃ x or ∀ x , for any variable x, then it is called **quantifier-free**.

An **existential formula** is a formula starting with a sequence of existential quantification followed by a quantifier-free formula.

formula

1. the set of **terms**
   a **variable**
   a **constant**
   $\mathbf{f(t_1,...,t_n)}$,

2. the **atomic formulas**.
   $\mathbf{t_1} = \mathbf{t_2}$
   $\mathbf{R(t_1,...,t_n)}$

3. the set of **formulas**
   ¬ ϕ
   (ϕ ∧ ψ), (ϕ ∨ ψ)
   ∃ x ϕ
   ∀ x ϕ

https://en.wikipedia.org/wiki/Well-formed_formula

Atomic sentences

A **sentence** is usually defined as
a **formula** without **free variables**.

An **atomic formula** is
a **formula** without **connectives**.

examples)

an **atomic formula** is P(x)
where x is a certain individual variable.
an **atomic sentence** is P(c)
where c is a certain predicate constant.

	atomic	
	$P(x)$	$P(x) \wedge Q(x)$
sentence	$P(c)$	$P(c) \wedge Q(c)$

formula :
$P(x)$, $P(x) \wedge Q(x)$, $P(c)$, $P(c) \wedge Q(c)$

sentence :
$P(c)$, $P(c) \wedge Q(c)$

atomic formula :
$P(x)$, $P(c)$

atomic sentence :
$P(c)$

https://www.quora.com/What-is-the-difference-between-an-atomic-sentence-and-an-atomic-formula-in-first-order-logic

Model and evaluation

There is a problem with **formuals** containing **free variables**:

to know whether they are **true**,
we need not only a **model**
(i.e. some **interpretation** of **predicate** and **functional constants**)
but also **evaluate** these variables.

this means that many such **formulas** are contingent
upon their **free variables** which can be undesirable.

https://www.quora.com/What-is-the-difference-between-an-atomic-sentence-and-an-atomic-formula-in-first-order-logic

Semantic interpretation of an atomic formula

The precise semantic interpretation of an atomic formula and an atomic sentence will vary from theory to theory.

- In propositional logic, atomic formulas are called propositional variables.[3] In a sense, these are nullary (i.e. 0-arity) predicates.
- In first-order logic, an atomic formula consists of a predicate symbol applied to an appropriate number of terms.

an **atomic formula** is P(x)
  where x is a certain individual variable.
an **atomic sentence** is P(c)
  where c is a certain predicate constant.

any **variable** is a **term**.
any **constant** is a **term**
an **n-ary function** expression $\mathbf{f(t_1,...,t_n)}$ is a **term**
  where $\mathbf{t_1},...,\mathbf{t_n}$ are terms

http://en.wikipedia.org/wiki/

Formal Language Interpretation

A formal language consists of a fixed collection of *sentences* (also called *words* or *formulas*, depending on the context) composed from a fixed set of *letters* or *symbols*. The inventory from which these letters are taken is called the *alphabet* over which the language is defined. To distinguish the strings of symbols that are in a formal language from arbitrary strings of symbols, the former are sometimes called *well-formed formulæ* (wff). The essential feature of a formal language is that its syntax can be defined without reference to interpretation. For example, we can determine that (*P* or *Q*) is a well-formed formula even without knowing whether it is true or false.

Example [edit]

A formal language $\mathcal{W}$ can be defined with the alphabet α = { △, □ }, and with a word being in $\mathcal{W}$ if it begins with △ and is composed solely of the symbols △ and □.

A possible interpretation of $\mathcal{W}$ could assign the decimal digit '1' to △ and '0' to □. Then △□△ would denote 101 under this interpretation of $\mathcal{W}$.

Alphabet

Letters / Symbols

Sentences / Formulas

Well Formed Formula

Syntax without interpretation

http://en.wikipedia.org/wiki/

Formal Language Expressions

The formal language used to create **expressions** consists of symbols

Symbols

- **constants**
  - **logical symbols**
  - **non-logical symbols**
- **variables**

https://en.wikipedia.org/wiki/Well-formed_formula

Logical Constants

T	true
F	false
¬	not
∧	and
∨	or
→	implies
∀	for all
∃	there exists
=	equals

https://en.wikipedia.org/wiki/Well-formed_formula

Non-logical Symbols

In case of a language of **first-order logic**
the **non-logical symbols**
- **predicates**
- **individual constants**

in an **interpretation**, symbols that may stand for
- **predicates**
- **individual constants**
- **variables**
- **functions**

the **logical symbols**
- **logical connectives**
- **quantifiers**
- **variables** that stand for **statements**

https://en.wikipedia.org/wiki/Well-formed_formula

Non-logical Symbols

A **non-logical symbol** only has meaning or semantic content
when one is assigned to it by means of an **interpretation**

A **sentence** containing a **non-logical symbol** lacks meaning
except under an **interpretation**

A **sentence** is said to be **true** or **false**
under an **interpretation**

The **logical constants** have the same meaning in all **interpretations**

https://en.wikipedia.org/wiki/Well-formed_formula

Symbols

A **logical symbol** is a fundamental concept in logic, tokens of which may be marks or a configuration of marks which form a particular pattern.[*citation needed*] Although the term "symbol" in common use refers at some times to the idea being symbolized, and at other times to the marks on a piece of paper or chalkboard which are being used to express that idea; in the formal languages studied in mathematics and logic, the term "symbol" refers to the idea, and the marks are considered to be a token instance of the symbol. [*dubious – discuss*] In logic, symbols build literal utility to illustrate ideas.

Symbols of a formal language need not be symbols *of* anything. For instance there are logical constants which do not refer to any idea, but rather serve as a form of punctuation in the language (e.g. parentheses). Symbols of a formal language must be capable of being specified without any reference to any interpretation of them.

A symbol or string of symbols may comprise a well-formed formula if it is consistent with the formation rules of the language.

A formal symbol as used in first-order logic may be a variable (member from a universe of discourse), a constant, a function (mapping to another member of universe) or a predicate (mapping to T/F).

Formal symbols are usually thought of as purely syntactic structures, composed into larger structures using a formal grammar, though sometimes they may be associated with an interpretation or model (a formal semantics).

http://en.wikipedia.org/wiki/

Interpretations for proposition logic

The formal language for propositional logic consists of formulas built up from propositional symbols (also called sentential symbols, sentential variables, and propositional variables) and logical connectives. The only non-logical symbols in a formal language for propositional logic are the propositional symbols, which are often denoted by capital letters. To make the formal language precise, a specific set of propositional symbols must be fixed.

The standard kind of interpretation in this setting is a function that maps each propositional symbol to one of the truth values true and false. This function is known as a *truth assignment* or *valuation* function. In many presentations, it is literally a truth value that is assigned, but some presentations assign truthbearers instead.

1. the set of **terms**
   - a **variable**
   - a **constant**
   - $\mathbf{f(t_1,\ldots,t_n)}$,
2. the **atomic formulas**.
   - $\mathbf{t_1 = t_2}$
   - $\mathbf{R(t_1,\ldots,t_n)}$
3. the set of **formulas**
   - $\neg\ \phi$
   - $(\ \phi \land \psi\ ),\ (\ \phi \lor \psi\ )$
   - $\exists\ x\ \phi$
   - $\forall\ x\ \phi$

http://en.wikipedia.org/wiki/

Interpretations for first-order logic

An example of interpretation $\mathcal{I}$ of the language **L** described above is as follows.

- Domain: A chess set
- Individual constants: a: The white King b: The black Queen c: The white King's pawn
- F(x): x is a piece
- G(x): x is a pawn
- H(x): x is black
- I(x): x is white
- J(x, y): x can capture y

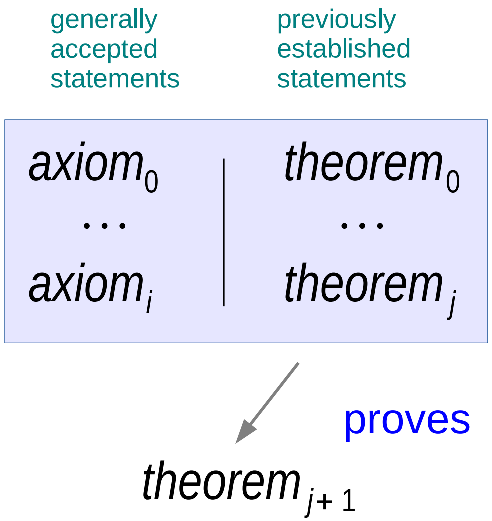
In the interpretation $\mathcal{I}$ of L:

- the following are true sentences: F(a), G(c), H(b), I(a) J(b, c),
- the following are false sentences: J(a, c), G(a).

http://en.wikipedia.org/wiki/

A **formal system** is broadly defined as
any well-defined system of abstract thought
based on the model of mathematics.

In mathematics, a **theorem** is a statement that has been proven on the basis of previously established statements,
such as other theorems, and generally accepted statements, such as axioms.



a **tautology** (from the Greek word ταυτολογία) is
a formula which is true in every possible interpretation.

An **axiom**, or postulate, is
a premise or starting point of reasoning.

As classically conceived,
an axiom is a premise so evident
as to be accepted as true without controversy.

http://en.wikipedia.org/wiki/

The WFF of **propositional logic**

(1) An **atomic proposition** is A is a **wff**

(2) If A and B, and C are **wffs**,
then so are $\neg$A, (A $\wedge$ B), (A $\vee$ B), (A $\rightarrow$ B), and (A $\leftrightarrow$ B).

(3) If A is a **wff**, then so is **(**A**)**.

http://en.wikipedia.org/wiki/

The WFF of **propositional logic**

(1) True and False are **wff**s.

(2) Each propositional **constant** (i.e. specific proposition), and
each propositional **variable** (i.e. a variable representing propositions) are **wff**s.

(3) Each **atomic formula** (i.e. a specific predicate with variables) is a **wff**.

(4) If A and B are **wff**s, then so are **¬**A, (A ∧ B), (A ∨ B), (A → B), and (A ↔ B).

(5) If x is a variable (representing objects of the universe of discourse), and
A is a wff, then so are ∃**x** A and ∀x A .

http://en.wikipedia.org/wiki/

First Order Logic and WFF

from en.wikipedia.org

Not all **strings** can represent **propositions** of the predicate logic.
Those which produce a **proposition**
when their symbols are **interpreted**
must follow the rules given below,
and they are called **wffs** of the first order predicate logic.

A predicate name followed by a list of variables such as P(x, y), where P is a predicate name, and x and y are variables, is called an atomic formula.

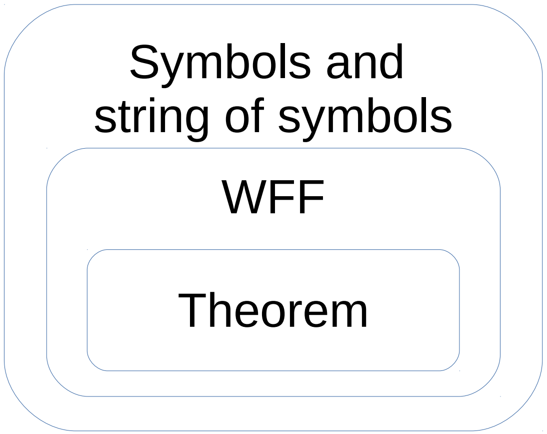
http://en.wikipedia.org/wiki/

WFF and Interpretation

Although the term "**formula**" may be used for **written marks** (for instance, on a piece of paper or chalkboard), it is more precisely understood **as the sequence** being expressed, with the **marks** being **a token instance of formula**.

It is **not necessary** for the existence of a formula that there be any actual **tokens** of it.

A **formal language** may thus have an infinite number of formulas regardless whether each formula has a **token instance**. Moreover, a single formula may have more than one **token instance**, if it is written more than once.



http://en.wikipedia.org/wiki/

Formulas are quite often interpreted as **propositions** (as, for instance, in propositional logic).

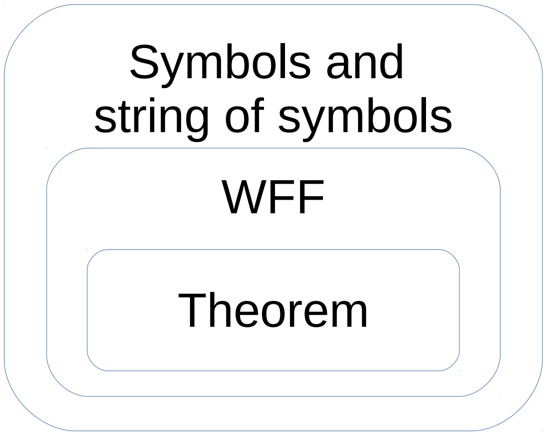
However formulas are **syntactic entities**, and as such must be specified in a formal language without regard to any interpretation of them.

An **interpreted formula** may be

- the name of something,
- an adjective,
- an adverb,
- a preposition,
- a phrase,
- a clause,
- an imperative sentence,
- a string of sentences,
- a string of names, etc.

A formula may even turn out to be **nonsense**, if the symbols of the language are specified so that it does.

Furthermore, a formula need not be given any interpretation.



http://en.wikipedia.org/wiki/

Proposition

proposition (*countable and uncountable*, *plural* **propositions**)

1. (*uncountable*) The act of offering (an idea) for consideration.
2. (*c* Appendix:Glossary ea or a plan offered. [quotations ▼]
3. (*countable*, *business settings*) The terms of a transaction offered.
4. (*countable*, *US*, *politics*) In some states, a proposed statute or constitutional amendment to be voted on by the electorate.
5. (*countable*, *logic*) The content of an assertion that may be taken as being true or false and is considered abstractly without reference to the linguistic sentence that constitutes the assertion.
6. (*countable*, *mathematics*) An assertion so formulated that it can be considered true or false.
7. (*countable*, *mathematics*) An assertion which is provably true, but not important enough to be called a theorem.
8. A statement of religious doctrine; an article of faith; creed. [quotations ▼]

   *the **propositions** of Wyclif and Huss*

9. (*poetry*) The part of a poem in which the author states the subject or matter of it.

http://en.wikipedia.org/wiki/

Symbols

A symbol is an idea, abstraction or concept, tokens of which may be marks or a configuration of marks which form a particular pattern. Symbols of a formal language need not be symbols of anything. For instance there are logical constants which do not refer to any idea, but rather serve as a form of punctuation in the language (e.g. parentheses). A symbol or string of symbols may comprise a **well-formed formula** if the formulation is **consistent** with the formation **rules** of the language. Symbols of a formal language must be capable of being specified without any reference to any interpretation of them.

Formal language

A formal language is a **syntactic entity** which consists of a set of finite strings of symbols which are its **words** (usually called its well-formed formulas). Which strings of symbols are words is determined by fiat by the creator of the language, usually by specifying a set of formation rules. Such a language can be defined without reference to any meanings of any of its expressions; it can exist before any interpretation is assigned to it – that is, before it has any meaning.

Formation rules

Formation rules are a **precise description** of **which** strings of symbols are the well-formed formulas of a formal language. It is synonymous with the set of strings over the alphabet of the formal language which constitute well formed formulas. However, it does not describe their semantics (i.e. what they mean).

http://en.wikipedia.org/wiki/

Syntax (2)

Propositions

A proposition is a **sentence** expressing something **true** or **false**. A proposition is identified ontologically as an idea, concept or abstraction whose token instances are patterns of symbols, marks, sounds, or strings of words. Propositions are considered to be syntactic entities and also truthbearers.

Formal theories

A formal theory is a **set of sentences** in a formal language.

Formal systems

A formal system (also called a **logical calculus**, or a **logical system**) consists of a **formal language** together with a **deductive apparatus** (also called a deductive system). The deductive apparatus may consist of a set of **transformation rules** (also called inference rules) or a set of **axioms**, or have both. A formal system is used to derive one expression from one or more other expressions. Formal systems, like other syntactic entities may be defined without any interpretation given to it (as being, for instance, a system of arithmetic).

Syntactic consequence within a formal system

A formula A is a syntactic consequence within some formal system ***FS*** of a set Γ of formulas if there is a derivation in formal system ***FS*** of A from the set Γ.

$\Gamma \vdash_{FS} A$

Syntactic consequence does not depend on any interpretation of the formal system.

Syntactic completeness of a formal system

A formal system ***S*** is **syntactically complete** (also deductively complete, maximally complete, negation complete or simply complete) iff for each formula A of the language of the system either A or ¬A is a theorem of ***S***. In another sense, a formal system is syntactically complete iff no unprovable axiom can be added to it as an axiom without introducing an inconsistency. Truth-functional **propositional logic** and **first-order predicate logic** are **semantically complete**, but **not** syntactically complete (for example the propositional logic statement consisting of a single variable "a" is not a theorem, and neither is its negation, but these are not tautologies).

Interpretations

An interpretation of a formal system is the assignment of meanings to the symbols, and truth values to the sentences of a formal system. The study of interpretations is called **formal semantics**. Giving an interpretation is synonymous with constructing a model. An interpretation is expressed in a metalanguage, which may itself be a formal language, and as such itself is a syntactic entity.

Satisfiability and Validity

from en.wikipedia.org

In mathematical logic, satisfiability and validity are elementary concepts of semantics.

A formula is **satisfiable** if it is possible to find **an** interpretation (model) that makes the formula true. **some** S are P

A formula is **valid** if **all** interpretations make the formula true. **every** S is a P

A formula is **unsatisfiable** if **none** of the interpretations make the formula true. **no** S are P

A formula is **invalid** if some such interpretation makes the formula false. **some** S are **not** P

These four concepts are related to each other in a manner exactly analogous to Aristotle's square of opposition.

a theory is **satisfiable** if **one** of the interpretations makes each of the axioms of the theory **true**.

a theory is **valid** if **all** of the interpretations make each of the axioms of the theory **true**.

a theory is **unsatisfiable** if **all** of the interpretations make each of the axioms of the theory **false**.

a theory is **invalid** if **one** of the interpretations makes each of the axioms of the theory **false**.

http://en.wikipedia.org/wiki/

Reduction of Validity to Unsatisfiability

For classical logics,

can reexpress the **validity** of a formula to **satisfiability**,

because of the relationships between the concepts expressed in the square of opposition.

In particular φ is **valid** if and only if ¬φ is **unsatisfiable**,

which is to say it is not true that ¬φ is satisfiable.

Put another way, φ is **satisfiable** if and only if ¬φ is **invalid**.

http://en.wikipedia.org/wiki/

a tautology (from the Greek word ταυτολογία) is a formula which is true in every possible interpretation.

In logic, semantic completeness is the converse of soundness for formal systems.

A formal system is "**semantically complete**" when all its **tautologies** are **theorems**

A formal system is "**sound**" when all **theorems** are **tautologies**

semantically complete

every tautology → theorem

sound

every theorem → tautology

(that is, they are **semantically valid** **formulas**: formulas that are true under every interpretation of the language of the system that is **consistent** with the rules of the system).

A formal system is **consistent** if for all formulas φ of the system, the formulas φ and ¬φ (the negation of φ) are not both theorems of the system (that is, they cannot be both proved with the rules of the system).

http://en.wikipedia.org/wiki/

Soundness

from en.wikipedia.org

An argument is **sound** if and only if

- The argument is **valid**.
- All of its premises are **true**.

For instance,

All men are mortal. (true)
Socrates is a man. (true)
Therefore, Socrates is mortal. (sound)

The argument is valid
(because the conclusion is true based on the premises, that is, that the conclusion follows the premises) and since the premises are in fact true, the argument is **sound**.

The following argument is valid but not sound:

All organisms with wings can fly. (false)
Penguins have wings. (true)
Therefore, penguins can fly. (valid)

Since the first premise is actually false, the argument, though valid, is not sound.

http://en.wikipedia.org/wiki/

Soundness and Completeness

The crucial properties of this set of rules are that they are sound and complete. Informally this means that the rules are correct and that no other rules are required.

http://en.wikipedia.org/wiki/

Sound, Complete

There are many deductive systems for first-order logic that are sound (all provable statements are true in all models) and complete (all statements which are true in all models are provable). Although the logical consequence relation is only semidecidable, much progress has been made in automated theorem proving in first-order logic. First-order logic also satisfies several metalogical theorems that make it amenable to analysis in proof theory, such as the Löwenheim–Skolem theorem and the compactness theorem.

Sound – all *provable* statements are **true** in all models

Complete – all statements which are *true* in all models are **provable**

http://en.wikipedia.org/wiki/

Turnstile

In mathematical logic and computer science the symbol $\vdash$ has taken the name **turnstile** because of its resemblance to a typical turnstile if viewed from above. It is also referred to as **tee** and is often read as "yields", "proves", "satisfies" or "entails". The symbol was first used by Gottlob Frege in his 1879 book on logic, *Begriffsschrift*.[1]

Martin-Löf analyzes the $\vdash$ symbol thus: "...[T]he combination of Frege's Urteilsstrich, judgement stroke [|], and Inhaltsstrich, content stroke [—], came to be called the assertion sign."[2] Frege's notation for a judgement of some content A

$$\vdash A$$

can be then be read

I know A is true".[3]

In the same vein, a conditional assertion

$$P \vdash Q$$

can be read as:

From P, I know that Q

In logic, the symbol $\vDash$, $\vDash$ or $\models$ is called the **double turnstile**. It is closely related to the turnstile symbol $\vdash$, which has a single bar across the middle. It is often read as "entails", "models", "is a semantic consequence of" or "is stronger than".[1] In TeX, the turnstile symbols $\vDash$ and $\models$

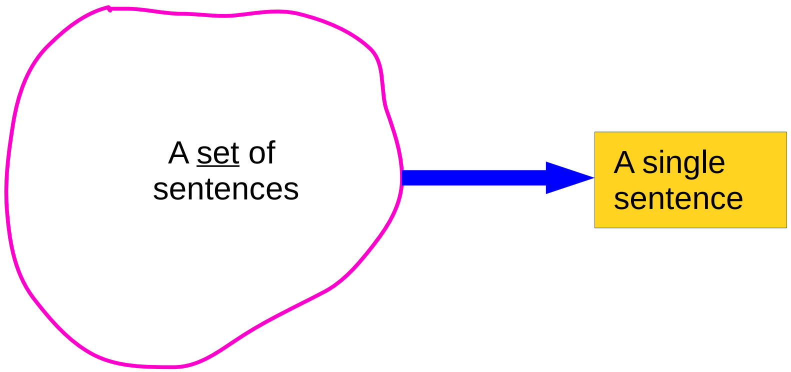
http://en.wikipedia.org/wiki/

Double Turnstile

In logic, the symbol ⊨, ⊨ or ⊨ is called the **double turnstile**. It is closely related to the turnstile symbol ⊢, which has a single bar across the middle. It is often read as "entails", "models", "is a semantic consequence of" or "is stronger than".[1] In TeX, the turnstile symbols ⊨ and ⊨

The double turnstile is a binary relation. It has several different meanings in different contexts:

- To show semantic consequence, with a set of sentences on the left and a single sentence on the right, to denote that if every sentence on the left is true, the sentence on the right must be true, e.g. $\Gamma \models \varphi$. This usage is closely related to the single-barred turnstile symbol which denotes syntactic consequence.
- To show satisfaction, with a model (or truth-structure) on the left and a set of sentences on the right, to denote that the structure is a model for (or satisfies) the set of sentences, e.g. $\mathcal{A} \models \Gamma$.
- To denote a tautology, $\models \varphi$. which is to say that the expression φ is a semantic consequence of the empty set.



http://en.wikipedia.org/wiki/

Premise

A **premise** : an assumption that something is true.

an **argument** requires

a set of (at least) **two** declarative sentences ("propositions")
known as the **premises**

along with **another** declarative sentence ("proposition")
known as the **conclusion**.

From Middle English, from Old French premisse, from Medieval Latin premissa ("set before") (premissa propositio ("the proposition set before")), feminine past participle of Latin praemittere ("to send or put before"), from prae- ("before") + mittere ("to send").

two premises and **one conclusion** :
the basic **argument** structure

Because all men are mortal and Socrates is a man,
Socrates is mortal.

2 premises
1 conclusion

3 propositions

Valid Argument Forms (Propositional)

Modus ponens (MP)

If A, then B
A
Therefore, B

Modus tollens (MT)

If A, then B
Not B
Therefore, not A

Hypothetical syllogism (HS)

If A, then B
If B, then C
Therefore, if A, then C

Disjunctive syllogism (DS)

A or B
Not A
Therefore, B

Modus ponens
(Latin) “the way that affirms by affirming"

Modus tollens
(Latin) "the way that denies by denying"

Syllogism
(Greek: συλλογισμός syllogismos) – "conclusion," "inference"

Modus Ponens

The Prolog resolution algorithm
based on the modus ponens form of inference

a general **rule** – the major premise and
a specific **fact** – the minor premise

All men are mortal	rule
Socrates is a man	fact
Socrates is mortal	

modus ponendo ponens
(Latin) “the way that affirms by affirming";
often abbreviated to **MP** or **modus ponens**

P implies Q;
P is asserted to be true,
so therefore Q must be true

one of the accepted mechanisms for the construction of deductive proofs
that includes the "rule of definition" and the "rule of substitution"

Facts	a	a
Rules	a → b	b :- a
Conclusion	b	b

Facts	man(’Socrates’).
Rules	mortal(X) :- man(X).
Conclusion	mortal(’Socrates’).

Modus Ponens (revisited)

Facts	a	a	minor term
Rules	a → b	b :- a	major term
Conclusion	b	b	

Syllogism : etymology

syllogism (*plural* **syllogisms**)

1. (*logic*) An inference in which one proposition (the conclusion) follows necessarily from two other propositions, known as the premises. [quotations ▼]
1. (*obsolete*) A trick, artifice.

Etymology [edit]

Wikipedia has an article **syllogism**

From Old French *silogisme* ("syllogism"), from Latin *syllogismus*, from Ancient Greek συλλογισμός (*sullogismós*, "inference, conclusion").

http://en.wikipedia.org/wiki/

Syllogism (1)

A syllogism (Greek: συλλογισμός – syllogismos – "conclusion," "inference") is

a kind of logical argument that applies deductive reasoning to arrive at a conclusion
based on two or more **propositions** that are asserted or assumed to be true.

In its earliest form, defined by Aristotle,
from the combination of
a general statement (the major premise) and ⟺ rule
a specific statement (the minor premise), ⟺ fact
a conclusion is deduced.

For example, knowing
that all men are mortal (major premise) and ⟺ rule
that Socrates is a man (minor premise), ⟺ fact
we may validly conclude that Socrates is mortal.

Syllogism (2)

A categorical syllogism consists of three parts:

Major premise:	All humans are mortal.	⟺	major term	(the predicate of the conclusion)
Minor premise:	All Greeks are humans.	⟺	minor term	(the subject of the conclusion)
Conclusion:	All Greeks are mortal.			

Each part - a categorical proposition - two categorical terms

In Aristotle, each of the premises is in the form

"All A are B"	universal proposition
"Some A are B"	particular proposition
"No A are B"	universal proposition
"Some A are not B"	particular proposition

Each of the premises has one term in common with the conclusion:
this common term is called
a major term in a major premise (the predicate of the conclusion)
a minor term in a minor premise (the subject of the conclusion)

Mortal is the major term,
Greeks is the minor term.
Humans is the middle term

Derivation

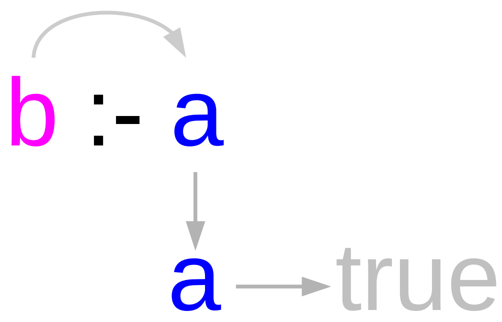
A **reversed modus ponens** is used in Prolog

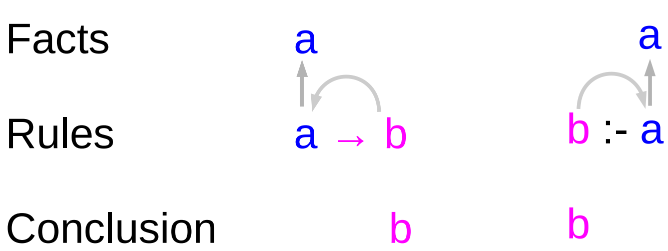
Prolog tries to prove that
a query (b) is a consequence of
the database content (a, a ⇒ b).

Using the major premise, it goes from b to a,
and using the minor premise, from a to true.

Such a sequence of goals is called a **derivation**.

A derivation can be **finite** or **infinite**.





Facts	a
Rules	b :- a
Conclusion	b

Horn Clause

the **resolvent** of **two Horn clauses** is itself **a Horn clause**
the **resolvent** of **a goal clause** and **a definite clause** is **a goal clause**

These properties of Horn clauses can lead to greater efficiencies in proving a theorem (represented as the negation of a goal clause).

Propositional Horn clauses are also of interest in computational complexity, where the problem of finding truth value assignments to make a conjunction of **propositional Horn clauses** true is a **P-complete** problem (in fact solvable in linear time), sometimes called **HORNSAT**. (The **unrestricted Boolean satisfiability** problem is an **NP-complete** problem however.) **Satisfiability** of **first-order Horn clauses** is undecidable.

By iteratively applying the resolution rule, it is possible

- to tell whether a propositional formula is **satisfiable**
- to prove that a first-order formula is unsatisfiable;

- this method may prove the **satisfiability** of a first-order formula,
- but not always, as it is the case for all methods for first-order logic

References

[1] http://en.wikipedia.org/
[2]