Angle Recoding 2. Wu
2. AR (Angle Recode)

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① AR [Hu]

skip certain micro rotations

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$\mu(i) = 0 \rightarrow$ skip

desire to minimize

$$\sum_{i=0}^{N} |\mu(i)|$$

so that the total number of CORDIC iterations can be minimized

Angle Recoding ← Multiplier Recoding

angle recoding method for efficient
implementation of the CORDIC algorithm
Hu & Naganathan, ISCAS 89

Greedy algorithm

the angle quantization error

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$

skipping rotations

$\mu(i) = 0$

null operations

$$\begin{cases} x(i+1) = x(i) \\ y(i+1) = y(i) \end{cases}$$

keep the iteration number N the same

in the AQ perspective, remove these steps
and use different index variable

"index variable conversion"

the iteration number N becomes N'

the number of subangles N_A of EAS

$N_A \Leftarrow N'$ not fixed but varying

Optimization Problem

given a rotation angle θ

find rotation sequence $\mu(i) \in \{-1, 0, +1\}$

for $0 \leq i \leq N-1$

Such that angle quantization error

$$|\xi_{m,AR}| < a(N-1)$$

and the effective iteration number

$$N' = \boxed{\sum_{i=0}^{N} |\mu(i)|} \quad \text{is minimized}$$

$\theta(0) = \theta$, $\{\mu(i) = 0,\ 0 \leq i \leq N-1\}$, $k = 0$.

repeat until $|\theta(k)| < a(N-1)\ D_0$

choose i_k, $0 \leq i_k \leq N-1$

$$\big|\,|\theta(k)| - a(i_k)\big| = \underset{0 \leq i \leq N-1}{\mathrm{Min}} \big|\,|\theta(k)| - a(i_k)\big|$$

$$\theta(k+1) = \theta(k) - \mu(i_k)\, a(i_k)$$

$$\mu(i_k) = \mathrm{Sign}(\theta(k))$$

AQ & AR

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$ N AR

$$= \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)}\right) \right]$$ N' AQ

$$= \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j) \right]$$ N'

$$N' \triangleq \sum_{i=0}^{N-1} |\mu(i)|$$ the effective transition number

count non-zero rotation

$s(j) \in \{0, 1, \cdots, N-1\}$ the rotational sequence

determines the micro-rotation angle in the j-th iteration

$\alpha(j) \in \{-1, 0, +1\}$ the directional sequence

$\{-1, +1\}$ (after sampling) controls the direction of the j-th micro-rotation of $a(s(j))$

$\tilde{\theta}(j)$ the jth micro-rotation angle

$$\tilde{\theta}(j) = \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)}\right)$$

AR essentially tries to approximate θ with the combination of selected angle elements from a pre-defined **elementary angle set** (EAS).

the EAS consists of **all** possible values of $\tilde{\theta}(j)$'s

the EAS S_1 in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, \quad s^* \in \{0, 1, \cdots, N-1\} \}$$

$$\begin{aligned} \xi_{m,AR} &\equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \\ &= \theta - \left[\sum_{j=0}^{N'} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right] \\ &= \theta - \left[\sum_{j=0}^{N'} \tilde{\theta}(j) \right] \end{aligned}$$

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A - 1} \theta_i$$

AR performs AQ of the target angle θ

the sub-angle θ_i becomes $\tilde{\theta}(i) = \tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$

$$N_A \longrightarrow \boxed{N' \triangleq \sum_{i=0}^{N-1} |\mu(i)|}$$

the effective transition number

given a rotation angle θ

AR

find rotation sequence $\mu(i) \in \{-1, 0, +1\}$ for $0 \leq i \leq N-1$

AQ

find the combination of elementary angles from S_1

such that angle quantization error

$$|\xi_{m,AR}| < a(N-1)$$

and the effective iteration number

$$N' = \sum_{i=0}^{N} |\mu(i)| \quad \text{is minimized}$$

EAS S_1 AR

elementary angle	value
r(1) = atan(-2^{-0})	
r(2) = atan(-2^{-1})	
r(3) = atan(-2^{-2})	
r(4) = atan(-2^{-3})	
r(5) = atan(-2^{-4})	
r(6) = atan(-2^{-5})	
r(7) = atan(-2^{-6})	
r(8) = atan(-2^{-7})	
r(9) = atan(0)	
r(10) = atan(2^{-7})	
r(11) = atan(2^{-6})	
r(12) = atan(2^{-5})	
r(13) = atan(2^{-4})	
r(14) = atan(2^{-3})	
r(15) = atan(2^{-2})	
r(16) = atan(2^{-1})	
r(17) = atan(2^{-0})	

N_A (r(1) to r(17))

N conventional CORDIC (r(9) to r(17))

```
>> mu = [1, 0, 0, -1, 0, 0, -1, -1, 0, 0, 0, 1, 0, 0, 0, 1]
>> length(mu)
ans =  16
>> s =[ 0: 15]
>> atan(1)
ans =  0.78540
>> pi/4
ans =  0.78540
>> sum(atan(2.^(-s)) .* mu)
ans =  0.63813
>> 13 * pi / 32
ans =  1.2763
```

i	Conventional CORDIC	only {-1, +1}	Angle Recode	skip allowed {-1, 0, +1}
0	S(0) = 0	α(0) = +1	S(0) = 0	α(0) = +1
1	S(1) = 1	α(1) = -1	S(1) = 1	α(1) = 0
2	S(2) = 2	α(2) = +1	S(2) = 2	α(2) = 0
3	S(3) = 3	α(3) = +1	S(3) = 3	α(3) = -1
4	S(4) = 4	α(4) = -1	S(4) = 4	α(4) = 0
5	S(5) = 5	α(5) = +1	S(5) = 5	α(5) = 0
6	S(6) = 6	α(6) = -1	S(6) = 6	α(6) = -1
7	S(7) = 7	α(7) = -1	S(7) = 7	α(7) = -1
8	S(8) = 8	α(8) = +1	S(8) = 8	α(8) = 0
9	S(9) = 9	α(9) = -1	S(9) = 9	α(9) = 0
10	S(10) = 10	α(10) = -1	S(10) = 10	α(10) = 0
11	S(11) = 11	α(11) = +1	S(11) = 11	α(11) = +1
12	S(12) = 12	α(12) = +1	S(12) = 12	α(12) = 0
13	S(13) = 13	α(13) = -1	S(13) = 13	α(13) = 0
14	S(14) = 14	α(14) = +1	S(14) = 14	α(14) = 0
15	S(15) = 15	α(15) = +1	S(15) = 15	α(15) = +1

i	Angle Recode			removing skipped angle	
0	$S(0) = 0$	$\alpha(0) = +1$	0	$S(0) = 0$	$\alpha(0) = +1$
1	$S(1) = 1$	$\alpha(1) = 0$	1		
2	$S(2) = 2$	$\alpha(2) = 0$	2		
3	$S(3) = 3$	$\alpha(3) = -1$	3	$S(1) = 3$	$\alpha(1) = -1$
4	$S(4) = 4$	$\alpha(4) = 0$	4		
5	$S(5) = 5$	$\alpha(5) = 0$	5		
6	$S(6) = 6$	$\alpha(6) = -1$	6	$S(2) = 6$	$\alpha(2) = -1$
7	$S(7) = 7$	$\alpha(7) = -1$	7	$S(3) = 7$	$\alpha(3) = -1$
8	$S(8) = 8$	$\alpha(8) = 0$	8		
9	$S(9) = 9$	$\alpha(9) = 0$	9		
10	$S(10) = 10$	$\alpha(10) = 0$	10		
11	$S(11) = 11$	$\alpha(11) = +1$	11	$S(4) = 11$	$\alpha(4) = +1$
12	$S(12) = 12$	$\alpha(12) = 0$	12		
13	$S(13) = 13$	$\alpha(13) = 0$	13		
14	$S(14) = 14$	$\alpha(14) = 0$	14		
$W-1 = 15$	$S(15) = 15$	$\alpha(15) = +1$	15	$S(5) = 15$	$\alpha(5) = +1$
				$N'-1$	

i (W)	Conventional CORDIC	only {-1, +1}	Angle Quant (N′)	skip allowed {-1, 0, +1}
0	$S(0) = 0$	$\alpha(0) = +1$	$S(0) = 0$	$\alpha(0) = +1$
1	$S(1) = 1$	$\alpha(1) = -1$	$S(1) = 3$	$\alpha(1) = -1$
2	$S(2) = 2$	$\alpha(2) = +1$	$S(2) = 6$	$\alpha(2) = -1$
3	$S(3) = 3$	$\alpha(3) = +1$	$S(3) = 7$	$\alpha(3) = -1$
4	$S(4) = 4$	$\alpha(4) = -1$	$S(4) = 11$	$\alpha(4) = +1$
5	$S(5) = 5$	$\alpha(5) = +1$	$S(5) = 15$	$\alpha(5) = +1$
6	$S(6) = 6$	$\alpha(6) = -1$		
7	$S(7) = 7$	$\alpha(7) = -1$		
8	$S(8) = 8$	$\alpha(8) = +1$		
9	$S(9) = 9$	$\alpha(9) = -1$		
10	$S(10) = 10$	$\alpha(10) = -1$		
11	$S(11) = 11$	$\alpha(11) = +1$		
12	$S(12) = 12$	$\alpha(12) = +1$		
13	$S(13) = 13$	$\alpha(13) = -1$		
14	$S(14) = 14$	$\alpha(14) = +1$		
15	$S(15) = 15$	$\alpha(15) = +1$		