Graph Coloring (9A)

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Graph Coloring

graph coloring is a special case of graph labeling;

it is an assignment of labels (colors)
to elements of a graph subject to certain constraints.

a **vertex coloring**
is a way of coloring the vertices of a graph
such that no two adjacent vertices share the same color

an **edge coloring**
assigns a color to each edge so that no two adjacent
edges share the same color

a **face coloring** of a planar graph
assigns a color to each face or region so that no two
faces that share a boundary have the same color.

https://en.wikipedia.org/wiki/Graph_coloring

Graph Coloring Relations

an **edge coloring** of a graph
is just a **vertex coloring** of its **line graph**,

a **face coloring** of a plane graph
is just a **vertex coloring** of its **dual graph**.

However, non-vertex coloring problems
are often stated and studied as is.

a graph coloring means almost always a **vertex coloring**.

Since a vertex with a loop could never be properly colored, a **loopless** graph is generally assumed.

https://en.wikipedia.org/wiki/Graph_coloring

k-coloring and chromatic number

k-coloring
a coloring using at most **k colors**

chromatic number, **χ(G)**
the smallest number of colors
needed to color a graph **G**

A graph that can be assigned a (proper) **k-coloring** is
k-colorable

A graph whose **chromatic number** is exactly **k** is
k-chromatic

https://en.wikipedia.org/wiki/Graph_coloring

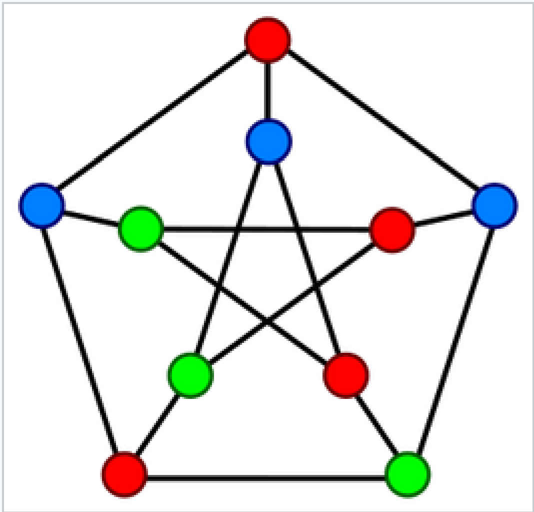
Color Class

A subset of vertices assigned to the same color is called a **color class**,

every such class forms an independent set.

a **k-coloring** is the same
as a **partition** of the vertex set
into **k** independent sets,

the terms **k-partite** and **k-colorable**
have the same meaning.



A proper vertex coloring of the Petersen graph with 3 colors, the minimum number possible.

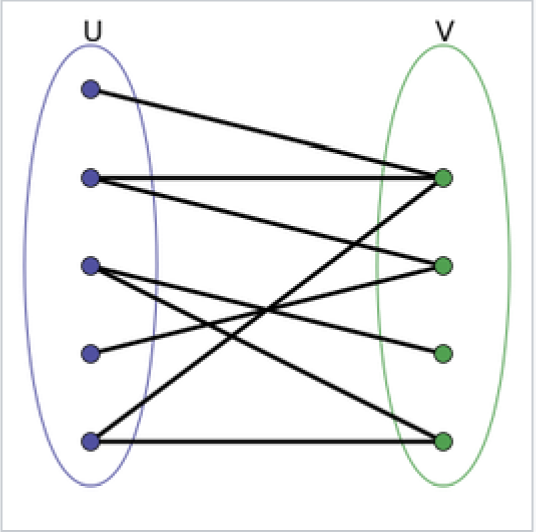
https://en.wikipedia.org/wiki/Graph_coloring

Bipartite Graph

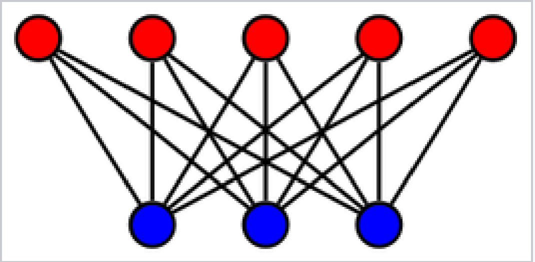
a bipartite graph (or bigraph) is a graph
whose vertices can be divided
into two disjoint and independent sets U and V
such that every edge connects
a vertex in U to one in V.

Vertex sets U and V are usually called
the parts of the graph.

Equivalently, a bipartite graph is
a graph that does not contain any **odd-length cycles**.



Example of a bipartite graph without cycles



A complete bipartite graph with m = 5 and n = 3

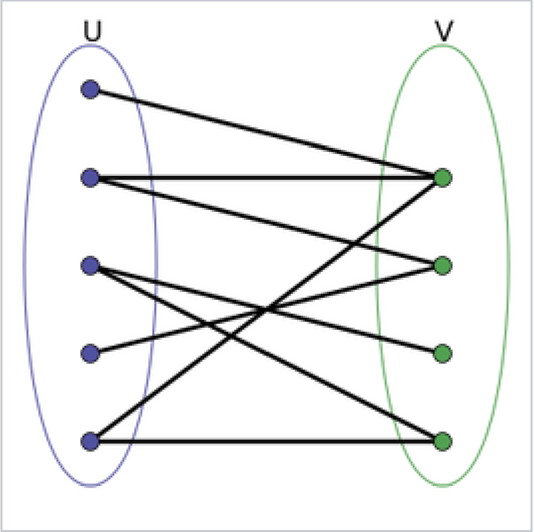
https://en.wikipedia.org/wiki/Bipartite_graph

Bipartite Graph : 2-colorable

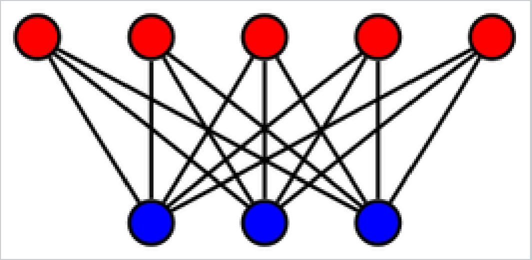
The two sets U and V may be thought of as a coloring of the graph with **two colors**:

if one colors all nodes in U blue,
and all nodes in V green,
each edge has endpoints of differing colors,
as is required in the graph coloring problem.

In contrast, such a coloring is impossible
in the case of a non-bipartite graph,
such as a triangle: 3 colors



Example of a bipartite graph without cycles



A complete bipartite graph with m = 5 and n = 3

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Bipartite Graph : degree sequence

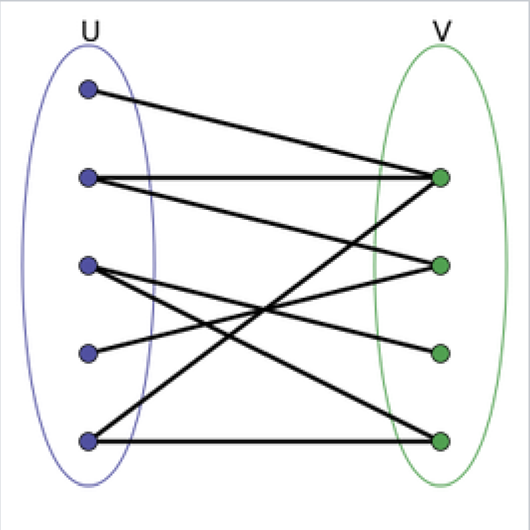
The degree sum formula
for a bipartite graph states that

$$\sum_{v \in V} \deg(v) = \sum_{u \in U} \deg(u) = |E| \,.$$

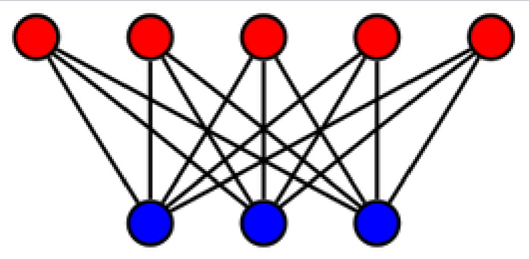
The degree sequence of a bipartite graph is the pair of lists each containing the degrees of the two parts U and V.

For example, the complete bipartite graph $K_{3,5}$ has degree sequence (5,5,5), (3,3,3,3,3)

$K_{5,3}$ has degree sequence (3,3,3,3,3), (5,5,5)



Example of a bipartite graph without cycles


A complete bipartite graph with m = 5 and n = 3

https://en.wikipedia.org/wiki/Bipartite_graph

References

[1] http://en.wikipedia.org/
[2]