Laurent Series and z-Transform - Geometric Series Double Pole Examples A

20180320-2

2 formulas of z

①
$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

②
$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$$\boxed{\frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \frac{-0.5z^2}{(z-1)(z-0.5)}}$$

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{-1}{(z^{-1}-1)(z^{-1}-2)} = \left(\frac{1}{z^{-1}-1} - \frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{z}{1-z} - \frac{z}{1-2z}\right)$$

$$= \left(\frac{-z}{z-1} + \frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-1}{z-1} + \frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-0.5z}{(z-1)(z-0.5)}\right)$$

$$= \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

(A) $f(z)$ (B) $X(z)$

(1) $$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

(1)-(A) $|z| < 1$ $$f(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$f(z)$ $|z| > 2$ $$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

(1)-(B) $|z| < 1$ $$X(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$X(z)$ $|z| > 2$ $$X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

(2) $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

(2)-(A) $|z| < 0.5$ $$f(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \qquad \boxed{1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$f(z)$ $|z| > 1$ $$f(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

(2)-(B) $|z| < 0.5$ $$X(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \qquad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$X(z)$ $|z| > 1$ $$X(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
Ⓐ $f(z)$	$\lvert z\rvert < p$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$	$\boxed{+1^{n-1}-2^{n-1}}$ $(n\geq 1)$
	$\lvert z\rvert > q$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$\boxed{-1^{n-1}+2^{n-1}}$ $(n<1)$
Ⓑ $X(z)$	$\lvert z\rvert < p$	$\boxed{-1^{n-1}+2^{n-1}}$ $(n<1)$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$
	$\lvert z\rvert > q$	$\boxed{+1^{n-1}-2^{n-1}}$ $(n\geq 1)$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$	$\boxed{+1^{n-1}-2^{n-1}}$ $(n\geq 1)$
	$X(z)$	$\boxed{-1^{n-1}+2^{n-1}}$ $(n<1)$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$
$\lvert z\rvert > q$	$f(z)$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$\boxed{-1^{n-1}+2^{n-1}}$ $(n<1)$
	$X(z)$	$\boxed{+1^{n-1}-2^{n-1}}$ $(n\geq 1)$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	causal $(n\geq 0)$	causal $(n\geq 1)$
$\lvert z\rvert > q$	$f(z)$	anticausal $(n<0)$	anticausal $(n<1)$
$\lvert z\rvert < p$	$X(z)$	anticausal $(n<1)$	anticausal $(n<0)$
$\lvert z\rvert > q$	$X(z)$	causal $(n\geq 1)$	causal $(n\geq 0)$

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	causal $(n\geq 0)$	causal $(n\geq 1)$
$\lvert z\rvert < p$	$X(z)$	anticausal $(n<1)$	anticausal $(n<0)$
$\lvert z\rvert > q$	$f(z)$	anticausal $(n<0)$	anticausal $(n<1)$
$\lvert z\rvert > q$	$X(z)$	causal $(n\geq 1)$	causal $(n\geq 0)$

① $\dfrac{-1}{(z-1)(z-2)} \overset{z^{-1}}{\longleftrightarrow}$ ② $\dfrac{-0.5z^{2}}{(z-1)(z-0.5)}$

$$\left(\frac{1}{z-1} - \frac{1}{z-2}\right) \qquad\qquad \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$\boxed{z}$ $\quad -\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z} \qquad\qquad \boxed{z}$ $\quad +\dfrac{z}{1-z} - \dfrac{z}{1-2z}$

$\boxed{|z|<1}$ $\quad |z|<1 \quad |0.5z|<1 \qquad\qquad \boxed{|z|<0.5}$ $\quad |z|<1 \quad |2z|<1$

$\boxed{z^{-1}}$ $\quad \dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}} \qquad\qquad \boxed{z^{-1}}$ $\quad -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$

$\boxed{|z|>2}$ $\quad |z^{-1}|<1 \quad |2z^{-1}|<1 \qquad\qquad \boxed{|z|>1}$ $\quad |z^{-1}|<1 \quad |0.5z^{-1}|<1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad\qquad +\frac{z}{1-z} - \frac{z}{1-2z}$$

$$\cdot\tfrac{1}{z}\Downarrow\Uparrow\cdot z \quad\quad \cdot\tfrac{2}{z}\Downarrow\Uparrow\cdot\tfrac{z}{2} \qquad\qquad \cdot\tfrac{1}{z}\Downarrow\Uparrow\cdot z \quad\quad \cdot\tfrac{1}{2z}\Downarrow\Uparrow\cdot 2z$$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad\qquad -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

① $\dfrac{-1}{(z-1)(z-2)}$ $\xleftrightarrow{z^{-1}}$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$\left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$ $\left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$\boxed{z}$ $-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$ $\boxed{z}$ $+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$

$\boxed{|z|<1}$ $f(z)$ causal $(n \geq 0)$ $\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ $\boxed{z^{-1}}$ $-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$

$\boxed{|z|>2}$ $\boxed{|z|>1}$

$X(z)$ causal $(n \geq 1)$ $X(z)$ causal $(n \geq 0)$

① $\dfrac{-1}{(z-1)(z-2)}$ $\quad\xleftrightarrow{z^{-1}}\quad$ ② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$\left(\dfrac{1}{z-1}-\dfrac{1}{z-2}\right)$ $\qquad$ $\left(-\dfrac{z}{(z-1)}+\dfrac{0.5z}{(z-0.5)}\right)$

$\boxed{z}$ $\quad -\dfrac{1}{1-z}+\dfrac{0.5}{1-0.5z}$

$\boxed{|z|<1}$ $\quad$ f(z) causal $(n \geq 0)$

X(z) anticausal $(n \leq 0)$

$\boxed{z}$ $\quad +\dfrac{z}{1-z}-\dfrac{z}{1-2z}$

$\boxed{|z|<0.5}$ $\quad$ f(z) causal $(n \geq 1)$

X(z) anticausal $(n \leq -1)$

$\boxed{z^{-1}}$ $\quad \dfrac{z^{-1}}{1-z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}}$

$\boxed{|z|>2}$ $\quad$ f(z) anticausal $(n \leq -1)$

X(z) causal $(n \geq 1)$

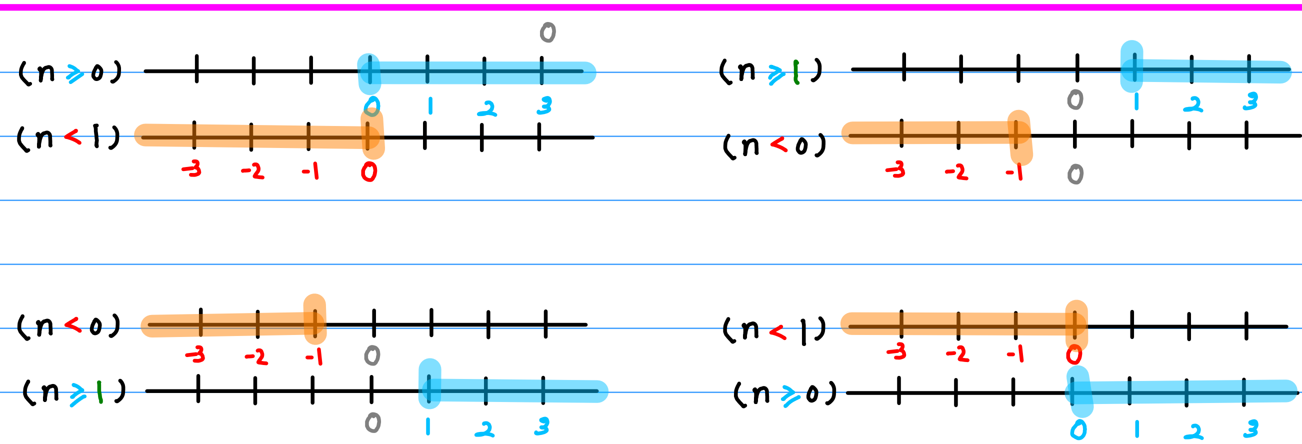
$\boxed{z^{-1}}$ $\quad -\dfrac{1}{1-z^{-1}}+\dfrac{0.5}{1-0.5z^{-1}}$

$\boxed{|z|>1}$ $\quad$ f(z) anticausal $(n \leq 0)$

X(z) causal $(n \geq 0)$

$(n \geq 0) \xrightarrow{-n} (n \leq 0) \equiv (n<1)$

$(n \geq 1) \xrightarrow{-n} (n \leq -1) \equiv (n<0)$



$$① \ \frac{-1}{(z-1)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$|z|<1$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2 z^1+\left(\tfrac{1}{2}\right)^3 z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$|z|<0.5$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] \quad -2^{n-1}$$

$|z|>2$

$$\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) = +\left[1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right] \quad -2^{n-1}$$

$*\ n = \quad 1 \quad 2 \quad 3$

$|z|>1$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)^1 z^0+\left(\tfrac{1}{2}\right)^2 z^{-1}+\left(\tfrac{1}{2}\right)^3 z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$*\ n = \quad 0 \quad 1 \quad 2$

$$① \ \frac{-1}{(z-1)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$|z| < 1$

$$-\frac{1}{1 - z} + \frac{0.5}{1 - 0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$2 = \left(\tfrac{1}{2}\right)^{-1}$$
$$\left(\tfrac{1}{2}\right) = 2^{-1}$$

$$X(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right] \quad -1^{n-1}$$
$$+\left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right] \quad +2^{n-1}$$

$n = \quad 0 \quad -1 \quad -2$

$|z| < 0.5$

$$+\frac{z}{1 - z} - \frac{z}{1 - 2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] \quad -2^{n-1}$$

$$X(z) = +\left[\left(\tfrac{1}{1}\right)^{-1} z^1 + \left(\tfrac{1}{1}\right)^{-2} z^2 + \left(\tfrac{1}{1}\right)^{-3} z^3 + \cdots\right] \quad +1^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$n = \quad -1 \quad -2 \quad -3$

$|z| > 2$

$$\frac{z^{-1}}{1 - z^{-1}} - \frac{z^{-1}}{1 - 2z^{-1}}$$

$$2 = \left(\tfrac{1}{2}\right)^{-1}$$
$$\left(\tfrac{1}{2}\right) = 2^{-1}$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^{-1} z^{-2} + \left(\tfrac{1}{1}\right)^{-2} z^{-3} + \cdots\right] \quad +1^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z) = +\left[1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right] \quad +1^{n-1}$$
$$-\left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right] \quad -2^{n-1}$$

$n = \quad 1 \quad 2 \quad 3$

$|z| > 1$

$$-\frac{1}{1 - z^{-1}} + \frac{0.5}{1 - 0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] \quad -1^{n-1}$$
$$+\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] \quad +2^{n-1}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] \quad -1^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$n = \quad 0 \quad 1 \quad 2$

VI

$$① \ \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} ② \ \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$|z|<1$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} + \left(\tfrac{1}{1}\right)^{-2} z^1 + \left(\tfrac{1}{1}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

$|z|<0.5$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{1}\right)^{-1} z^1 + \left(\tfrac{1}{1}\right)^{-2} z^2 + \left(\tfrac{1}{1}\right)^{-3} z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$|z|>2$

$$\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[\left(\tfrac{1}{1}\right)^0 z^{-1} + \left(\tfrac{1}{1}\right)^{-1} z^{-2} + \left(\tfrac{1}{1}\right)^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$X(z) = +\left[1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{a_n = +1^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$|z|>1$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$\mathrm{ROC}(z) \quad f(z) \Longleftrightarrow a_n \quad \mathrm{RNG}(n)$

$|z| < p \qquad n \geq 0$

I $\quad \mathrm{ROC}(z^{-1}) \quad f(z) \Longleftrightarrow -a_n \quad \overline{\mathrm{RNG}}(n)$

$|z| > \frac{1}{p} \qquad n < 0$

$\mathrm{ROC}(z) \quad f(z) \Longleftrightarrow a_n \quad \mathrm{RNG}(n)$

$|z| < p \qquad n \geq 0$

II $\quad \mathrm{ROC}(z^{-1}) \quad f(z^{-1}) \Longleftrightarrow a_{-n} \quad \mathrm{RNG}(-n)$

$|z| > \frac{1}{p} \qquad n < 1$

$\mathrm{ROC}(z) \quad f(z) \Longleftrightarrow a_n \quad \mathrm{RNG}(n)$

$|z| < p \qquad n \geq 0$

III $\quad \mathrm{ROC}(z) \quad f(z^{-1}) \Longleftrightarrow -a_{-n} \quad \ll \mathrm{RNG}(n) \gg \quad$ I + II

$|z| < p \qquad n \geq 1$

$\mathrm{ROC}(z) \quad f(z) \Longleftrightarrow a_n \quad \mathrm{RNG}(n)$

$|z| < p \qquad n \geq 0$

IV $\quad \mathrm{ROC}(z) \quad X(z) \Longleftrightarrow a_{-n} \quad \mathrm{RNG}(-n)$

$|z| < p \qquad n < 1$

$$\text{Ⅲ} = \text{Ⅰ} + \text{Ⅱ}$$

Ⅲ

$ROC(z)$	$f(z^{-1})$	$\Longleftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$	Ⅰ + Ⅱ
$\lvert z\rvert < p$				$n \geqslant 1$	

$ROC(z)$	$f(z)$	$\Longleftrightarrow$	a_n	$RNG(n)$
$\lvert z\rvert < p$				$n \geqslant 0$

Ⅰ

$ROC(z^{-1})$	$f(z)$	$\Longleftrightarrow$	$-a_n$	$\overline{RNG}(n)$
$\lvert z\rvert > \frac{1}{p}$				$n < 0$

$n \leqslant -1$

Ⅱ

$ROC(z)$	$f(z^{-1})$	$\Longleftrightarrow$	$-a_{-n}$	$\overline{RNG}(-n)$
$\lvert z\rvert < p$				$n \geqslant 1$

Compare (I) with (IV)

ROC(z)	f(z)	$\Longleftrightarrow$	a_n	RNG(n)
$\lvert z\rvert < p$				$n \geq 0$

	ROC(z)	f(z)	$\Longleftrightarrow$	a_n	RNG(n)
(I)	$ROC(z^{-1})$	$f(z)$	$\Longleftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\lvert z\rvert > \frac{1}{p}$				$n < 0$
				-1	complement
(IV)	ROC(z)	$X(z)$	$\Longleftrightarrow$	a_{-n}	RNG(−n)
	$\lvert z\rvert < p$				$n < 1$
				$-n$	$-n$
					symmetrical

Ⅰ

ROC(z^{-1})	$f(z)$ $\Longleftrightarrow$	$-a_n$	$\overline{\text{RNG}}(n)$
$\lvert z\rvert > \frac{1}{p}$			$n < 0$

$(p,q)=(1,2)$ $\quad$ $(p,q)=(0.5,1)$

		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$	$+1^{n-1}-2^{n-1}$ $(n\geq 1)$
z^{-1}		-1	-1
$\lvert z\rvert > q$	$f(z)$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-1^{n-1}+2^{n-1}$ $(n<1)$

$(p,q)=(1,2)$ $\quad$ $(p,q)=(0.5,1)$

		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$X(z)$	$-1^{n-1}+2^{n-1}$ $(n<1)$	$+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$
z^{-1}		-1	-1
$\lvert z\rvert > q$	$X(z)$	$+1^{n-1}-2^{n-1}$ $(n\geq 1)$	$-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$

Ⅱ

$ROC(z^{-1})$	$f(z^{-1})$	a_{-n}	$RNG(-n)$
$\lvert z\rvert > \frac{1}{p}$			$n < 1$

z^{-1}

		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$f(z)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$
		$-n$	$-n$
$\lvert z\rvert > q$	$f(z)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$
		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$

z^{-1}

z^{-1}

		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$\lvert z\rvert < p$	$X(z)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$
		$-n$	$-n$
$\lvert z\rvert > q$	$X(z)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$
		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$

z^{-1}

$$2^{-n+1} = \left(\tfrac{1}{2}\right)^{n} \cdot 2 = \left(\tfrac{1}{2}\right)^{n-1} \qquad \left(\tfrac{1}{2}\right)^{-n-1} = 2^{n} \cdot 2 = 2^{n+1}$$

$$\left(\tfrac{1}{2}\right)^{-n+1} = 2^{n} \cdot \tfrac{1}{2} = 2^{n-1} \qquad 2^{-n-1} = \left(\tfrac{1}{2}\right)^{n} \cdot \tfrac{1}{2} = \left(\tfrac{1}{2}\right)^{n+1}$$

III	ROC(z) $|z| < p$	$f(z^{-1})$ $\xleftrightarrow{}$ $-a_{-n}$	RNG(−n) $n \geq 1$	I + II

z^{-1} $\longleftrightarrow$

		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$|z| < p$	$f(z)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$
		$\xleftrightarrow{-n,\ -1}$	
$|z| > q$	$f(z)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$
		$\xleftrightarrow{-n,\ -1}$	

$(p, q) = (1, 2)$ $(p, q) = (0.5, 1)$

z^{-1} $\longleftrightarrow$

		① $\dfrac{-1}{(z-1)(z-2)}$	② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$
$|z| < p$		$\xleftrightarrow{-n,\ -1}$	
	$X(z)$	$-1^{n-1} + 2^{n-1}$ $(n<1)$	$+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n<0)$
$|z| > q$		$\xleftrightarrow{-n,\ -1}$	
	$X(z)$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$	$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

$(p, q) = (1, 2)$ $(p, q) = (0.5, 1)$

IV	ROC(z)	X(z) ⟷	a_{-n}	RNG(-n)
	$\|z\| < p$			$n < 1$

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\|z\| < p$	f(z)	$-1^{n+1} + (\frac{1}{2})^{n+1}$ $(n \geq 0)$	$-n$
	X(z)	$-1^{n-1} + 2^{n-1}$ $(n < 1)$	
$\|z\| > q$	f(z)	$+1^{n+1} - (\frac{1}{2})^{n+1}$ $(n < 0)$	$-n$
	X(z)	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$	

		$(p,q)=(1,2)$	$(p,q)=(0.5,1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$\|z\| < p$	f(z)	$-n$	$+1^{n-1} - 2^{n-1}$ $(n \geq 1)$
	X(z)		$+1^{n+1} - (\frac{1}{2})^{n+1}$ $(n < 0)$
$\|z\| > q$	f(z)	$-n$	$-1^{n-1} + 2^{n-1}$ $(n < 1)$
	X(z)		$-1^{n+1} + (\frac{1}{2})^{n+1}$ $(n \geq 0)$

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal $\quad$ anticausal

①-Ⓐ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$|z| < 1$

$$f(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$-(1^1 + 1^2 z + 1^3 z^2 + \cdots) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

n=0 n=1 n=2 $\quad$ n=0 n=1 n=2

$|z| > 2$

$$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) - (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$$

$$(1^0 z^{-1} + 1^{-1} z^{-2} + 1^{-2} z^{-3} + \cdots) - \left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right)$$

n=-1 n=-2 n=-3 $\quad$ n=-1 n=-2 n=-3

②-Ⓐ

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$|z| < 0.5$

$$f(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \qquad \boxed{1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$+(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z + 2^1 z^2 + 2^2 z^3 + \cdots)$$

n=1 n=2 n=3 $\quad$ n=1 n=2 n=3

$|z| > 1$

$$f(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \qquad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$-(1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots) + \left(\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$$-(1^{-1} z^0 + 1^{-2} z^{-1} + 1^{-3} z^{-2} + \cdots) + (2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots)$$

n=0 n=-1 n=-2 $\quad$ n=0 n=-1 n=-2

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal causal

① - Ⓑ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2} \right)$

$|z| < 1 \qquad X(z) = -\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z} \qquad \boxed{-1^{n-1} + 2^{n-1}} \quad (n<1)$

$-\left(1^1 z^0 + 1^2 z^1 + 1^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$

$-\left(1^{-1} z^0 + 1^{-2} z^1 + 1^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$|z| > 2 \qquad X(z) = \dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$

$+\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

② - Ⓑ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)} \right)$

$|z| < 0.5 \qquad X(z) = +\dfrac{z}{1-z} - \dfrac{z}{1-2z} \qquad \boxed{+1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$

$+\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right)$

$+\left(1^0 z + 1^{-1} z^2 + 1^{-2} z^3 + \cdots\right) - \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$|z| > 1 \qquad X(z) = -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}} \qquad \boxed{-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$

$-\left(1^1 z^0 + 1^2 z^{-1} + 1^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

Ⅰ

$$\boxed{\begin{array}{llll} ROC(z^{-1}) & f(z) \Longleftrightarrow -a_n & & RNG(n) \\ |z| > \frac{1}{p} & & & n < 0 \end{array}}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad X(z) \qquad a_n = \boxed{-(\frac{1}{2})^{n-1} + 2^{n-1}} \quad (n < 1)$

$|z| > 2$ $\quad X(z) \qquad b_n = \boxed{+(\frac{1}{2})^{n-1} - 2^{n-1}} \quad (n \geq 1)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \Rightarrow a_n + b_n = 0$

$a_n = -b_n$

ROC

$|z| < a$ $\quad X(z) = \dfrac{a}{1-az} = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \downarrow -1\cdot$

ROC'

$|z| > a^{-1}$ $\quad X(z) = -\dfrac{z^{-1}}{1-a^{-1}z^{-1}} = -\displaystyle\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -a^{n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k$$

$$\boxed{\frac{a}{1-az}} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \boxed{\frac{z}{1-az}} = \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$\boxed{-\frac{z^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad \boxed{-\frac{a^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^k$$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad\qquad \frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

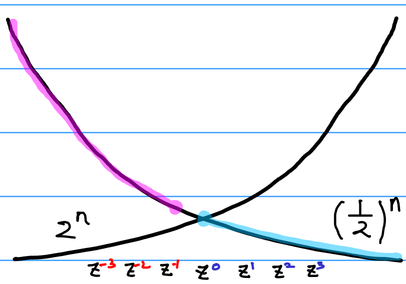
$$\| \qquad \| \qquad\qquad\qquad \| \qquad \|$$

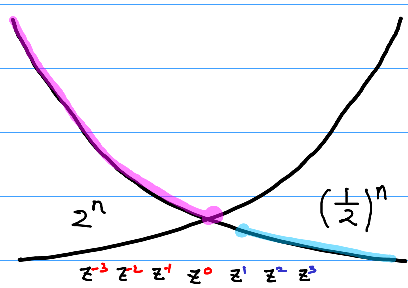
$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

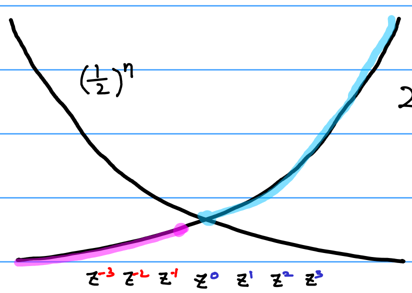
$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^{k} \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

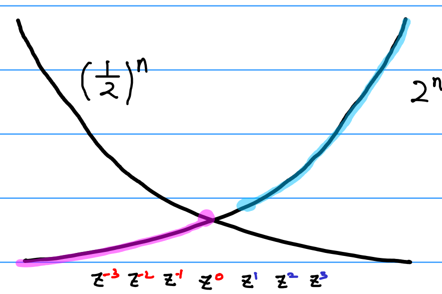
$$a + a^{2}z^{1} + a^{3}z^{2} + a^{4}z^{3} + \cdots \qquad\qquad z + az^{2} + a^{2}z^{3} + a^{3}z^{4} + \cdots$$

$$z^{-1} + a^{-1}z^{-2} + a^{-2}z^{-3} + a^{-3}z^{-4} + \cdots \qquad\qquad a^{-1} + a^{-2}z^{-1} + a^{-3}z^{-2} + a^{-4}z^{-3} + \cdots$$









Ⅳ $\boxed{\text{ROC}(z) \quad X(z) \longleftrightarrow a_{-n} \quad \text{RNG}(-n) \\ |z| < p \qquad\qquad n \le 0}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $(n \ge 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\quad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n \le 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\quad f(z) = \displaystyle\sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad a^{n+1}$ $\quad n \ge 0 \quad n \ge 1 \quad n < 0 \quad n < 1$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$$

$-n$

ROC $\quad X(z) = \displaystyle\sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\quad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$ $\quad n \le 0 \quad n \le 1 \quad n > 0 \quad n > 1$

$\boxed{\begin{array}{llll} ROC(z^{-1}) & f(z^{-1}) & a_{-n} & RNG(-n) \\ |z| > \frac{1}{p} & & & n < 1 \end{array}}$ (II)

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots] + [(\tfrac{1}{2}) + (\tfrac{1}{2})^2 z^1 + (\tfrac{1}{2})^3 z^2 + \cdots]$$

$$\boxed{a_n = -1^{n+1} + (\tfrac{1}{2})^{n+1} \quad (n \geq 0)}$$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -[(\tfrac{1}{1})^{-1} z^0 + (\tfrac{1}{1})^{-2} z^{-1} + (\tfrac{1}{1})^{-3} z^{-2} + \cdots] + [2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

ROC

$|z| < a$ $\quad f(z) = \dfrac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\Big\downarrow z^{-1} \qquad \Big\downarrow -n \qquad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

ROC'

$|z| > a^{-1}$ $\quad f(z^{-1}) = \dfrac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n} \qquad a^{-n+1} \qquad n < 1 \quad n < 0 \quad n \geq 0 \quad n \geq 0$

$$= \sum_{k=0}^{-\infty} a^{-k+1} z^{k} \qquad = \left(\frac{1}{a}\right)^{n-1}$$

$$|z| > 1 \qquad\qquad |z| > 2$$

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{1}{1 - 2z^{-1}} \qquad (|z| > 2)$$

$$a_n = 1^n + 2^n \qquad (n \geq 0)$$

$$X(z)$$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots) \qquad n = 0,\ 1,\ 2, \cdots$$

$$z^{-1} \qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots \qquad n-1$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \qquad n = 1,\ 2,\ 3, \cdots$$

$$X(z) = \frac{z^{-1}}{1 - z^{-1}} - \frac{z^{-1}}{1 - 2z^{-1}} \qquad (|z| > 2)$$

$$a_n = 1^{n-1} + 2^{n-1} \qquad (n \geq 1)$$

$$|z|<1 \qquad\qquad |z|<0.5$$

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \qquad \left(|z|<0.5\right)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots\right) - \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots\right) \qquad n = 0,\ 1,\ 2, \cdots$$

$f(z)$ $\quad z \quad$ $1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$ $\quad n-1$

$$\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^2 + 2^2 z^3 + \cdots\right) \qquad n = 1,\ 2,\ 3, \cdots$$

$$f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \qquad \left(|z|<0.5\right)$$

$$a_n = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$f(z) = \frac{\overset{|z|>1}{1}}{1-z} - \frac{\overset{|z|>2}{1}}{1-2z} \quad (|z|<0.5)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$f(z)$

$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots)$ $\quad n = 0, 1, 2, \cdots$

$z \quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots \quad n-1$

$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^{-1} + 2^1 z^2 + 2^2 z^3 + \cdots)$ $\quad n = 1, 2, 3, \cdots$

$$f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_n = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$a_n = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n<0)$$

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{1}{1 - 2z^{-1}} \quad (|z| > 2)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$$f(z) \quad \begin{array}{l} (1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots) \quad n = 0, 1, 2, \cdots \\ \downarrow z^{-1} \quad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots \quad \downarrow n-1 \\ (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n = 1, 2, 3, \cdots \end{array}$$

$$f(z) = \frac{z^{-1}}{1 - z^{-1}} - \frac{z^{-1}}{1 - 2z^{-1}} \quad (|z| > 2)$$

$$a_n = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

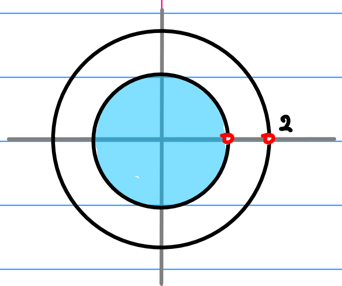
$$f(z) = \frac{z}{1 - z} - \frac{z}{1 - 2z} \quad (|z| < 0.5)$$

$$a_n = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

Causal $\quad f(z) \quad X(z)$

$|z| < 1 \qquad |z| > 2$

① – Ⓐ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$



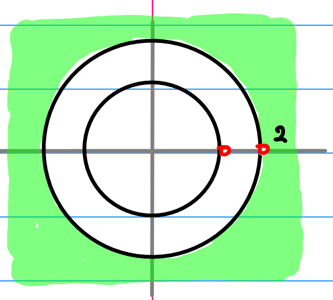
$|z|<1 \qquad |z|<2$

$$f(z) = (-1)\frac{1}{1-z} + \frac{0.5}{1-0.5z} \qquad (|z|<1)$$

$$a_n = (-1)\ 1^n + \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^n \qquad (n \geq 0)$$

$$-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

① – Ⓑ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$



$|z|>1 \qquad |z|>2$

$$X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$1^n - 2^n \qquad (n \geq 0)$$

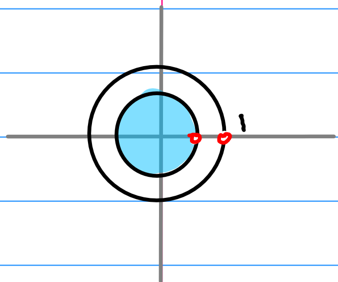
$$a_n = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

Causal $\quad f(z) \quad X(z)$

$|z| < 0.5 \quad |z| > 1$

②-Ⓐ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|<1 \qquad |z|<0.5$



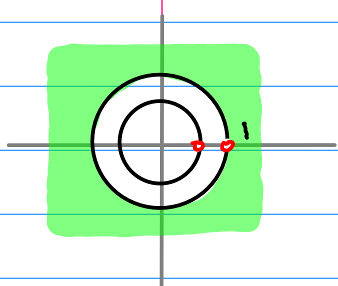
$f(z) = (+1)\dfrac{z}{1-z} - \dfrac{z}{1-2z} \quad (|z|<0.5)$

$1^n - 2^n \quad (n \geq 0)$

$a_n = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$

②-Ⓑ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|>1 \qquad |z|>0.5$



$X(z) = -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}} \quad (|z|>1)$

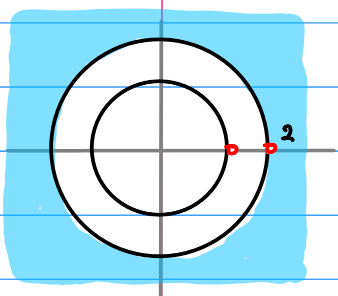
$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

Anti-causal

$f(z)$ $\quad |z| > 2$

$X(z)$ $\quad |z| < 1$

①-Ⓐ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$



$|z|>1 \qquad |z|>2$

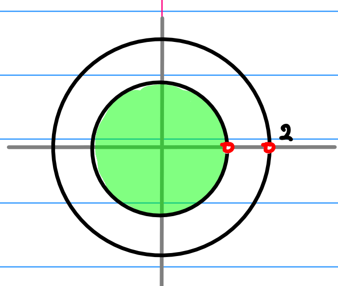
$$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$1^n - 2^n \quad (n \geq 0)$

$1^{n-1} - 2^{n-1} \quad (n \geq 1)$

$a_n = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n<0)$

①-Ⓑ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$



$|z|<1 \qquad |z|<2$

$$X(z) = -\frac{1}{1-z} + \frac{\frac{1}{2}}{1-\left(\frac{z}{2}\right)} \qquad (|z|<1)$$

$-1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

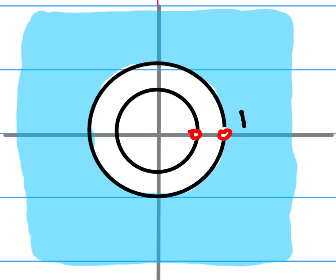
$a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)$

Anti-causal $f(z)$ $X(z)$

$|z| > 1$ $|z| < 0.5$

②-Ⓐ $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)} \right)$$

$|z|>1$ $|z|>0.5$



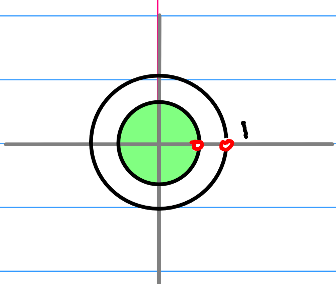
$$f(z) = (-1)\frac{1}{1-z^{-1}} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} \quad (|z|>1)$$

$$-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$a_n = -1^{n-1} + 2^{n-1} \quad (n<1)$$

②-Ⓑ $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)} \right)$$

$|z|<1$ $|z|<0.5$



$$X(z) = \frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<1)$$

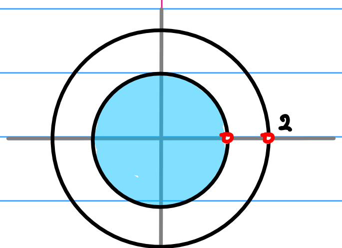
$$+1^{n} - 2^{n} \quad (n \geq 0)$$

$$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$a_n = 1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)$$

① - Ⓐ $\dfrac{-1}{(z-1)(z-2)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

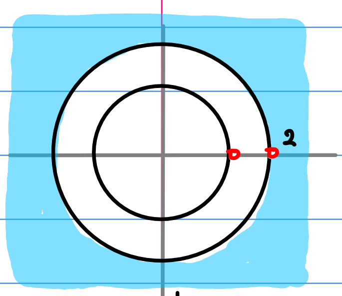


$|z| < 1$

$$f(z) = \boxed{\begin{aligned} &-\frac{(1)}{1-(z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(1)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(1)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$(n \geq 0)$ $a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$



$|z| > 2$

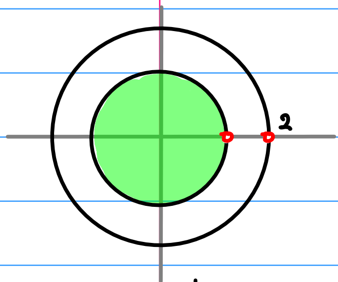
$$f(z) = \boxed{\begin{aligned} &\frac{(\frac{1}{z})}{1-(\frac{1}{z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}(1)^n\left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}(1)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}} \neq$$

$$= \sum_{n=-1}^{-\infty}(1)^{n+1} z^n - \sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n$$

$(n < 0)$ $a_n = 1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$

① - Ⓑ $\quad \dfrac{-1}{(z-1)(z-2)} = \boxed{X(z)}$ $\qquad |z| < 0.5$ anticausal $\qquad |z| > 2$ causal

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

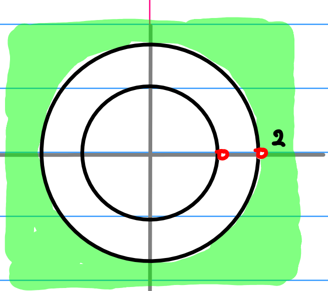


$|z| < 0.5$

$$\begin{aligned} X(z) &= \boxed{-\frac{(1)}{1-(z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})}} \\ &= -\sum_{n=0}^{\infty} (1)^{n+1} (z)^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} (z)^n \\ &= -\sum_{n=0}^{\infty} (1)^{n+1} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n \\ &= -\sum_{n=0}^{-\infty} (1)^{n-1} z^{-n} + \sum_{n=0}^{-\infty} (2)^{n-1} z^{-n} \end{aligned}$$

$(n \leq 0) \qquad a_n = -(1)^{n-1} + 2^{n-1}$

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$



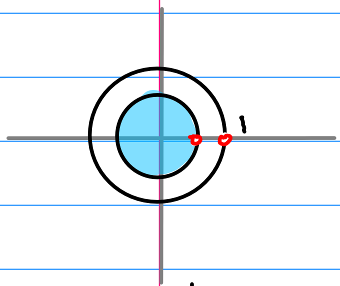
$|z| > 2$

$$\begin{aligned} X(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \qquad \neq \\ &= \sum_{n=0}^{\infty} (1)^n \left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty} (2)^n \left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty} (1)^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n} \end{aligned}$$

$(n > 0) \qquad a_n = (1)^{n-1} - (2)^{n-1}$

② - Ⓐ $\quad \dfrac{-0.5z^2}{(z-1)(z-0.5)} = \boxed{f(z)}$ $\quad |z| < 0.5$ causal $\quad |z| > 2$ anticausal

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = \left(\frac{z}{(1-z)} - \frac{z}{(1-2z)}\right)$$

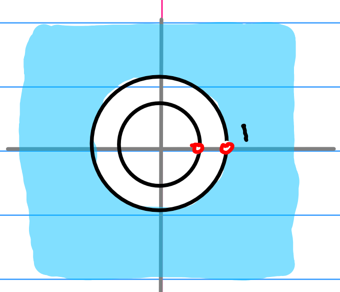


$|z| < 0.5$

$$\boxed{\begin{aligned} f(z) &= \frac{(z)}{1-(z)} - \frac{(z)}{1-(2z)} \\ &= \sum_{n=0}^{\infty} (1)^n (z)^{n+1} - \sum_{n=0}^{\infty} (2)^n (z)^{n+1} \\ &= \sum_{n=1}^{\infty} (1)^{n-1} z^n - \sum_{n=1}^{\infty} (2)^{n-1} z^n \end{aligned}} \quad \neq$$

$(n > 0) \quad a_n = 1^{n-1} - (2)^{n-1}$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = \left(\frac{z}{(1-z)} - \frac{z}{(1-2z)}\right)$$



$|z| > 2$

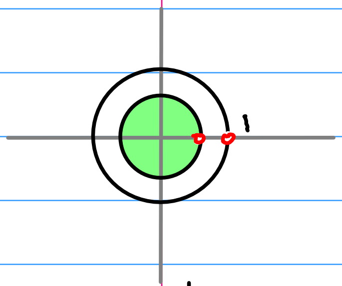
$$\boxed{\begin{aligned} f(z) &= -\frac{(1)}{1-(\frac{1}{z})} + \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} \\ &= -\sum_{n=0}^{\infty} (1)^{n+1} \left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} \left(\frac{1}{z}\right)^n \\ &= -\sum_{n=0}^{\infty} (1)^{n+1} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty} (1)^{n-1} z^n + \sum_{n=0}^{-\infty} (2)^{n-1} z^n$$

$(n \leq 0) \quad a_n = -1^{n-1} + (2)^{n-1}$

② - Ⓑ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = \left(\frac{z}{(1-z)} - \frac{z}{(1-2z)}\right)$$



$|z| < 0.5$

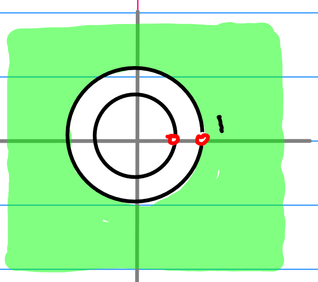
$$X(z) = \boxed{\begin{aligned} &+\frac{(z)}{1-(z)} - \frac{(z)}{1-(2z)} \\ &= \sum_{n=0}^{\infty}(1)^n(z)^{n+1} - \sum_{n=0}^{\infty}(2)^n(z)^{n+1} \\ &= \sum_{n=1}^{\infty}(1)^{n-1}z^n - \sum_{n=1}^{\infty}(2)^{n-1}z^n \end{aligned}}$$

$\neq$

$$= \sum_{n=-1}^{-\infty}(1)^{n+1}z^{-n} - \sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n}$$

$(n < 0)$ $a_n = (1)^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = \left(\frac{z}{(1-z)} - \frac{z}{(1-2z)}\right)$$



$|z| > 1$

$$X(z) = \boxed{\begin{aligned} &-\frac{(1)}{1-(\frac{1}{z})} + \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} \\ &= -\sum_{n=0}^{\infty}(1)^{n+1}\left(\tfrac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{z}\right)^n \\ &= -\sum_{n=0}^{\infty}(1)^{n+1}z^{-n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} \end{aligned}}$$

$(n \geq 0)$ $a_n = -(1)^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$