Monad P3 : Existential Types (1D)

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This document was produced by using LibreOffice.

Based on

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

Existential Quantification

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Existentials

Existential types, or

Existentials for short,

provide a way of

squashing a group of types

into one, single type.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Existentials

Existentials are part of GHC's type system **extensions**.

But not part of **Haskell98**

have to either compile with a command-line parameter of

-XExistentialQuantification,

or put at the top of your sources that use existentials.

{-# LANGUAGE ExistentialQuantification #-}

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

forall type variables

The forall keyword is to explicitly bring fresh **type variables** into scope

type variables

those variables that begin with a **owercase** letter

the compiler allows **any type** to fill these variables

those variables that are **universally quantified**

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

forall type variables

Example: A polymorphic function

map :: (a -> b) -> [a] -> [b]

a **lowercase type parameter**

implicitly begins with a **forall** keyword,

Example: Explicitly quantifying the type variables

map :: forall a b. (a -> b) -> [a] -> [b]

two type declarations for map are equivalent

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

forall type variables

Example: A polymorphic function

map :: (a -> b) -> [a] -> [b]

Example: Explicitly quantifying the type variables

map :: forall a b. (a -> b) -> [a] -> [b]

instantiating the general type of **map**

to a more specific type

a = Int

b = String

(Int -> String) -> [Int] -> [String]

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Hiding a type variable (1)

Normally when creating a new type

using **type**, **newtype**, **data**, etc.,

every **type variable** that appears on the right-hand side

must also appear on the left-hand side.

```
newtype ST s a = ST (State# s -> (# State# s, a #))
```

Existential types are a way of escaping

Existential types can be used for several different purposes.

But what they do is to **hide** a **type variable** on the right-hand side.

https://wiki.haskell.org/Existential_type

Hiding a type variable (2)

Normally, any type variable appearing on the right must

also appear on the left:

data Worker x y = Worker {buffer :: b, input :: x, output :: y}

This is an **error**, since the **type** b of the **buffer**

is not specified on the right

(b is a **type variable** rather than a **type**)

but also is not specified on the left

(there's no b in the left part).

In Haskell98, you would have to write

data Worker b x y = Worker {buffer :: b, input :: x, output :: y}

https://wiki.haskell.org/Existential_type

Hiding a type variable (3)

However, suppose that a **Worker** can use any type b
so long as it belongs to some particular **class**.
Then every **function** that uses a **Worker** will have a type like

```
foo :: (Buffer b) => Worker b Int Int
```

In particular, failing to write an **explicit type signature** **(Buffer b)**
will invoke the dreaded monomorphism restriction.

Using **existential types**, we can avoid this:

https://wiki.haskell.org/Existential_type

Hiding a type variable (4)

Using existential type :

```
data Worker x y =  forall b. Buffer b =>  Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

The **type** of the **buffer** (**Buffer**) now does not appear

in the **Worker** type at all. **Worker x y**

Explicit type signature :

```
data Worker b x y = Worker {buffer :: b, input :: x, output :: y}
foo :: (Buffer b) => Worker b Int Int
```

https://wiki.haskell.org/Existential_type

Hiding a type variable (5)

- it is now impossible for a function
  to demand a **Worker** having a specific type of **buffer**.

- the **type** of **foo** can now be derived automatically
  without needing an explicit **type signature**.
  (No **monomorphism** restriction.)

- since code now has no idea
  what **type** the buffer function returns,
  you are more limited in what you can do to it.

```
data Worker x y = forall b. Buffer b => Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

https://wiki.haskell.org/Existential_type

Hiding a type variable (6)

In general, when you use a **hidden type** in this way,

you will usually want that **type** to belong to a **specific class**,

or you will want to **pass some functions** along

that can work on that type.

Otherwise you'll have some value belonging

to a **random unknown type**,

and you won't be able to do anything to it!

https://wiki.haskell.org/Existential_type

Less specific types (1)

Note: You can use **existential types**

to **convert** a **more specific type**

into a **less specific one**.

constrained type variables

There is no way to perform the reverse conversion!

https://wiki.haskell.org/Existential_type

Less specific types (2)

This illustrates **creating a heterogeneous list**,
all of whose members implement "**Show**",
and progressing through that list to show these items:

```
data Obj = forall a. (Show a) => Obj a

    xs :: [Obj]
    xs = [Obj 1, Obj "foo", Obj 'c']

    doShow :: [Obj] -> String
    doShow [] = ""
    doShow ((Obj x):xs) = show x ++ doShow xs
```

With output: **doShow xs ==> "1\"foo\"'c'"**

https://wiki.haskell.org/Existential_type

Existentials in terms of **forall** (1)

It is also possible to express existentials with RankNTypes
as **type expressions** directly (without a **data** declaration)

forall r. (forall a. Show a => a -> r) -> r

(the leading forall r. is optional
unless the expression is part of another expression).

the equivalent type **Obj** :

data Obj = forall a. (Show a) => Obj a

https://wiki.haskell.org/Existential_type

Existentials in terms of **forall** (2)

The conversions are:

```
fromObj ::  Obj -> forall r. (forall a. Show a => a -> r) -> r
fromObj (Obj x) k = k x

toObj :: (forall r. (forall a. Show a => a -> r) -> r)  ->  Obj
toObj f = f Obj
```

https://wiki.haskell.org/Existential_type

Heterogeneous Lists

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Type hider

Suppose we have a group of values.

they may not be all the same **type**,

but they are all members of some **class**

thus, they have a certain **property**

It might be useful to throw all these values into a list.

normally this is impossible because lists elements

must be of the same type

(homogeneous with respect to types).

existential types allow us to loosen this requirement

by defining a **type hider** or **type box**:

```
data ShowBox = forall s. Show s => SB s
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (1)

```
data ShowBox = forall s. Show s => SB s                -- type hider
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

[SB (), SB 5, SB True] calls the **constructor**

on three values of different types,

to place them all into a single list

virtually the same type for each one.

Use the **forall** in the constructor

SB :: forall s. Show s => s -> ShowBox.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (2)

```
data ShowBox = forall s. Show s => SB s
heteroList :: [ShowBox]
heteroList = [SB (), SB 5, SB True]
```

When passing **heteroList** type parameters to a function

we cannot take out the **values** inside the **SB**

because their type might **Bool**. **Int**, **Char**, …

But each of the elements can be

converted to a **string** via **show**.

In fact, that's the only thing we know about them.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (3)

```
instance Show ShowBox where
 show (SB s) = show s

f :: [ShowBox] -> IO ()
f xs = mapM_ print xs

main = f heteroList
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Heterogeneous list example (4)

Example: Using our heterogeneous list

```
 instance Show ShowBox where
show (SB s) = show s        -- (*) see the comment in the text below
f :: [ShowBox] -> IO ()
f xs = mapM_ print xs
main = f heteroList
```

Example: Types of the functions involved

```
print :: Show s => s -> IO ()        -- print x = putStrLn (show x)
mapM_ :: (a -> m b) -> [a] -> m ()
mapM_ print :: Show s => [s] -> IO ()
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

mapM, **mapM_**, and **map** (1)

The core idea is that **mapM** maps
an "action" (ie function of type **a -> m b**) over a list and
gives you all the results as **m [b]**

mapM_ does the same thing,
but never collects the results, returning a **m ()**.

If you care about the results
of your **a -> m b** function, use **mapM**.
If you only care about the effect,
but not the resulting value,
use **mapM_**, because it can be more efficient

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

mapM, **mapM_**, and **map** (2)

Always use **mapM_** with functions of the type **a -> m ()**,

like **print** or **putStrLn**.

these functions return **()** to signify that only the effect matters.

If you used **mapM**, you'd get a list of **()** (ie **[(), (), ()]**),

which would be completely useless

but waste some memory.

If you use **mapM_**, you would just get a **()**,

but it would still print everything.

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

mapM, **mapM_**, and **map** (3)

Normal **map** is something different:

it takes a normal function **(a -> b)**

instead of one using a monad **(a -> m b)**.

This means that it cannot have any sort of effect

besides returning the changed list.

You would use it if you want to transform a list

using a normal function.

map_ doesn't exist because, since you don't have any effects,

you always care about the results of using **map**.

https://stackoverflow.com/questions/27609062/what-is-the-difference-between-mapm-and-mapm-in-haskell/27609146

Quantified types

as products and sums

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Quantified Types as Products and Sums

A **universally quantified type** may be interpreted

as an **infinite product** of **types**.

a **polymorphic function** can be understood

as a **product**, or a **tuple**, of **individual functions**,

one per every possible **type a**.

To construct a **value** of such **type**, we have

to provide all the **components** of the **tuple** at once.

-- one formula generating an infinity of functions

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Example: Identity function

```
id :: forall a. a -> a
id a = a
```

a **polymorphic function** can be understood

as a **product**, or a **tuple**, of **individual functions**,

one per every possible **type a**.

Int -> Int, Double -> Double, ...

Char -> Char, [Char] -> [Char], …

…

…

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

To construct a **value** of such **type**, we have

to provide all the **components** of the **tuple** at once.

in case of **numeric types**, one **numeric constant**

may be used to initialize **many types** at once.

Example: Polymorphic value

x :: forall a. Num a => a

x = 0

x may be conceptualized as a **tuple** consisting

of an **Int value**, a **Double value**, etc.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Similarly, an **existentially quantified type** may be interpreted as an **infinite sum**.

Example: Existential type

data ShowBox = forall s. Show s => SB s

may be conceptualized as a **sum**:

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantified Types as Products and Sums

Example: Existential type

data ShowBox = forall s. Show s => SB s

Example: Sum type

data ShowBox = SBUnit | SBInt Int | SBBool Bool | SBIntList [Int] | ...

to construct a **value** of this **type**,

we only have to pick one of the constructors.

A **polymorphic constructor SB**

combines all those constructors into one.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Quantification as a primitive

https://stackoverflow.com/questions/3071136/what-does-the-forall-keyword-in-haskell-ghc-do

Newtype creates a function (1)

newtype Parser a = Parser { parse :: String -> Maybe (a,String) }

1) A **type** named **Parser**.

2) A **term level constructor** of **Parser's** named **Parser**.

The **type** of this (constructor) function is

Parser :: (String -> Maybe (a, String)) -> Parser a

You give it a function of the type

(String -> Maybe (a, String))

and it wraps it inside a **Parser**

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (2)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
```

3) A **function** named **parse** to remove the **Parser** wrapper and get your function back. The type of this function is:

parse :: Parser a -> String -> Maybe (a, String)

A **term level constructor** named **Parser**

Parser :: (String -> Maybe (a, String)) -> Parser a

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (3)

```
Prelude> newtype
            Parser a = Parser { parse :: String -> Maybe (a,String) }

Prelude> :t Parser
Parser :: (String -> Maybe (a, String)) -> Parser a

Prelude> :t parse
parse :: Parser a -> String -> Maybe (a, String)
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (4)

newtype Parser a = Parser { parse :: String -> Maybe (a,String) }

the **term level constructor** (**Parser**)

the **function** to remove the wrapper (**parse**)

Both can have arbitrary names

No need to match the type name.

It's common to write:

newtype Parser a = Parser { *unParser* :: String -> Maybe (a,String) }

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (5)

newtype Parser a = Parser { unParser :: String -> Maybe (a,String) }

this name makes it clear **unParser** removes
the **wrapper** around the parsing function.

unParser :: Parser a -> String -> Maybe (a, String)

however, it is recommended that the **type** and **constructor**
have the same name when using **newtypes**.
(**Parser**, **Parser**)

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (6)

newtype Parser a = Parser { parser :: String -> Maybe (a,String) }

1) **Parser** is declared as a **type** with a **type parameter a**

2) can instantiate **Parser** by providing a **parser** function

p = Parser (\s -> Nothing)

3) a function name **parser** defined and

it is capable of *running Parser's*.

unwrap the function

then apply the function

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (7)

```
newtype Parser a = Parser { parser :: String -> Maybe (a,String) }

parser :: Parser a -> String -> Maybe (a, String)
parser (Parser  (\s -> Nothing)) "my input"
(\s -> Nothing)) "my input"
Nothing
```

You are unwrapping the function using **parse** and

then calling the unwrapped function with "myInput".

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (8)

First, let's have a look at a parser **newtype** without **record** syntax:

```
newtype Parser' a = Parser' (String -> Maybe (a,String))
```

it stores a **function String -> Maybe (a,String)**.

To run this parser, we will need to make a **new function:**

```
runParser' :: Parser' a -> String -> Maybe (a,String)
runParser' (Parser' f) i = f i
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (9)

```
runParser' :: Parser' a -> String -> Maybe (a,String)
runParser' (Parser' f) i = f i

runParser' (Parser' $ \s -> Nothing) "my input".
```

But now note that, since Haskell functions are curried,
we can simply remove the reference to the input **i** to get:

```
runParser'' :: Parser' -> (String -> Maybe (a,String))
runParser'' (Parser' f') = f'
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (10)

runParser'' :: Parser' -> (String -> Maybe (a,String))

runParser'' (Parser' f’) = f’

This function is exactly equivalent to **runParser'**,

but you could think about it differently:

instead of applying the parser function to the value explicitly,

it simply takes a parser and fetches the parser function from it;

(Parser' f’) → **f’**

however, thanks to **currying**, **runParser''**

can still be used with two arguments.

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (11)

```
newtype Parser a = Parser { parse :: String -> Maybe (a,String) }
newtype Parser' a = Parser' (String -> Maybe (a,String))
```

difference : record syntax with only one field

this record syntax automatically defines a function

```
parse :: Parser a  -> (String -> Maybe (a,String)),
```

which extracts the **String -> Maybe (a,String)** function
from the **Parser a**.

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Newtype creates a function (12)

newtype Parser a = Parser { parse :: String -> Maybe (a,String) }

parse can be used with two arguments thanks to **currying**,
and this simply has the effect of **running** the function stored
within the **Parser a**.

equivalent definition to the following code:

```
newtype Parser a = Parser (String -> Maybe (a,String))
```

```
parse :: Parser a -> (String -> Maybe (a,String))
parse (Parser p) = p
```

https://stackoverflow.com/questions/60291263/why-the-newtype-syntax-creates-a-function

Access functions in a record type (1)

```
data Person = Person {  firstName  :: String  ,
                        lastName   :: String  ,
                        age        :: Int     ,
                        height     :: Float   ,
                        phoneNo    :: String  ,
                        flavor     :: String
                     } deriving (Show)

ghci> :t flavor
flavor :: Person -> String
ghci> :t firstName
firstName :: Person -> String
```

return types of access functions

Person :: the input type of access functions

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Access functions in a record type (2)

```
data Car = Car String String Int deriving (Show)

ghci> Car "Ford" "Mustang" 1967
Car "Ford" "Mustang" 1967



data Car = Car {company :: String,
                model :: String,
                year :: Int} deriving (Show)

ghci> Car {company="Ford", model="Mustang", year=1967}
Car {company = "Ford", model = "Mustang", year = 1967}
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

Pair type example (1)

Universal quantification is useful

for defining **data types** that aren't already defined.

Suppose there was no such thing as **pairs** built into haskell.

Quantification could be used to define them.

```
{-# LANGUAGE ExistentialQuantification, RankNTypes #-}

newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)

makePair :: a -> b -> Pair a b
makePair a b = Pair $ \f -> f a b
```

Pair $ \f -> f a b :: Pair a b

f :: a -> b -> c

f a b :: c

f is not yet defined

c can be any type (**forall c**)

defining data type **c**

that aren't already defined

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (2)

```
newtype Pair a b = Pair (forall c. (a -> b -> c) -> c)
```

```
makePair :: a -> b -> Pair a b
makePair a b = Pair $ \f -> f a b
```

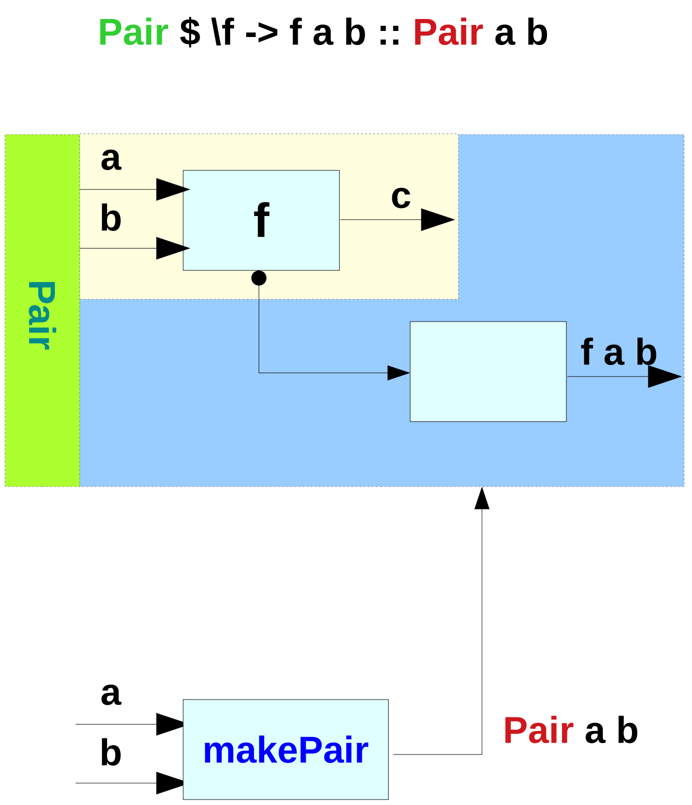
using a record type with a single field

```
λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
```

runPair is an access function

takes an input of the type **Pair a b**

returns an output of the type **forall c. (a -> b -> c) -> c**



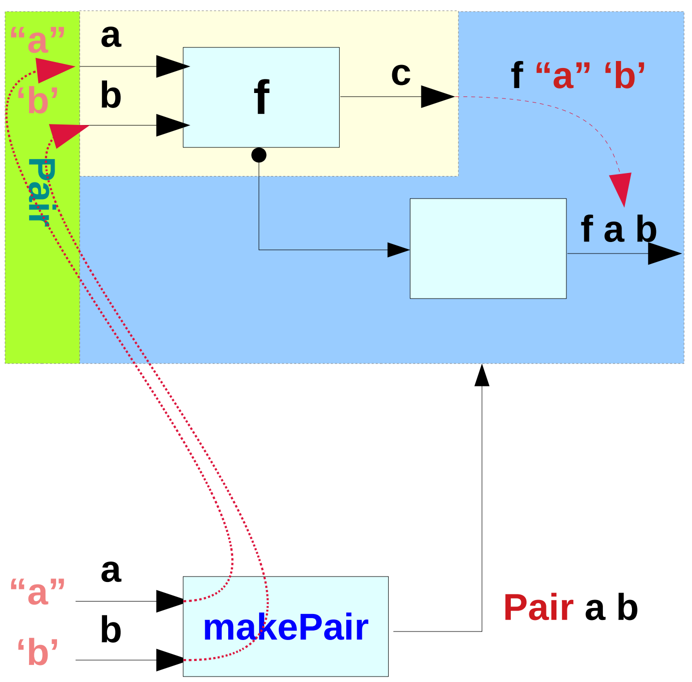
https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (3)

In GHCI

```
λ> :set -XExistentialQuantification
λ> :set -XrankNTypes
λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
λ> makePair a b = Pair $ \f -> f a b
λ> pair = makePair "a" 'b'

λ> :t pair
 pair :: Pair [Char] Char
λ> runPair pair (\x y -> x)     -- unwrap (a -> b -> c) -> c then apply
 "a"
λ> runPair pair (\x y -> y)    -- unwrap (a -> b -> c) -> c then apply
 'b'
```

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

Pair $ \f -> f "a" 'b' :: Pair a b

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (4)

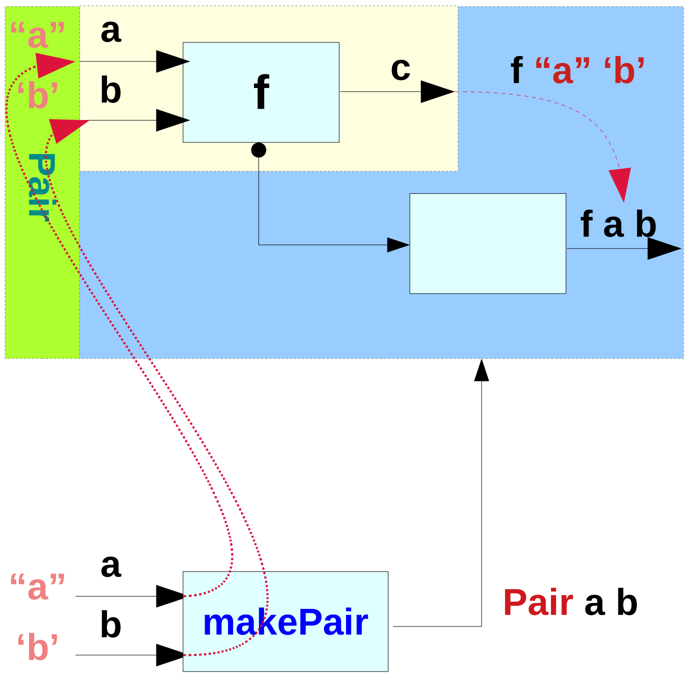
```
λ> newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}

λ> makePair a b = Pair $ \f -> f a b

λ> pair = makePair "a" 'b'
```

Pair $ \f -> f "a" 'b'

\f : function itself   f :: a -> b -> c

f "a" 'b' : the result of applying the function

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

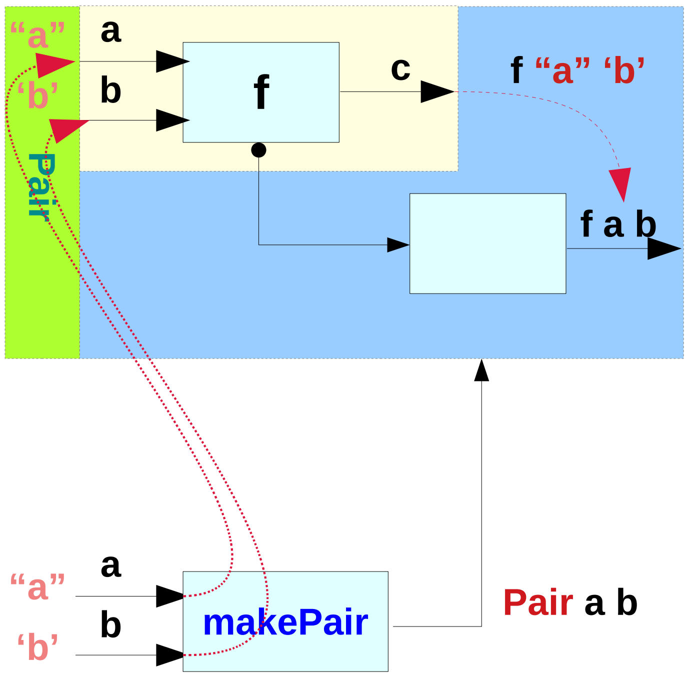
Pair $ \f -> f "a" 'b' :: Pair a b

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (5)

```
newtype Pair a b = Pair {runPair :: forall c. (a -> b -> c) -> c}
runPair ::  Pair a b -> forall c. (a -> b -> c) -> c

makePair a b = Pair $ \f -> f a b
runPair makePair a b = \f -> f a b              -- unwrapping

makePair "a" 'b'  = Pair $ \f -> f "a" 'b'
runPair makePair "a" 'b' = \f -> f "a" 'b'

pair = makePair                          :: Pair [Char] Char

runPair  pair  (\x y -> x) = (\x y -> x) "a" 'b'
runPair  pair  (\x y -> y) = (\x y -> y) "a" 'b'
```

Pair $ \f -> f a b :: Pair a b



makePair "a" 'b'

Pair $ \f -> f "a" 'b'   :: Pair a b

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

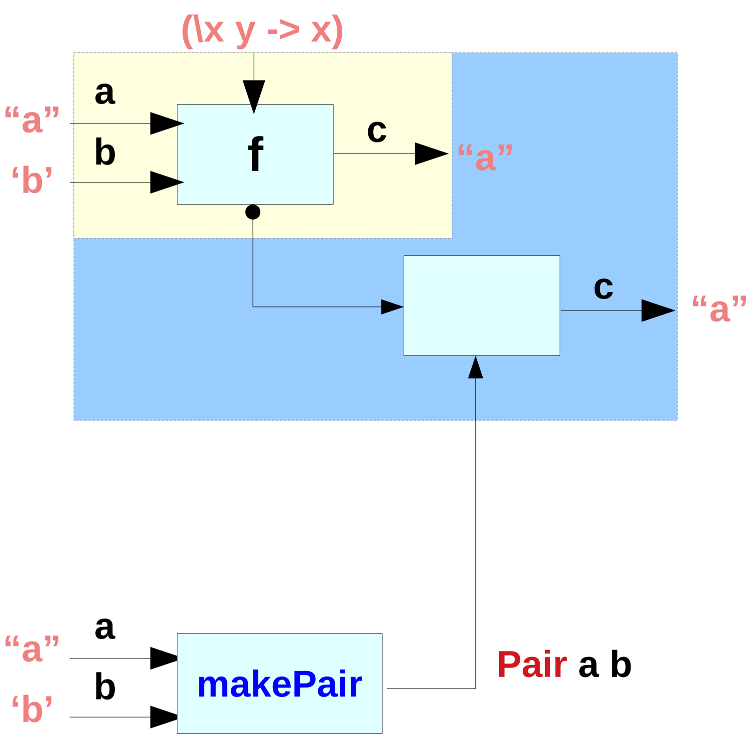
Pair type example (6)

```
runPair  pair  (\x y -> x) = (\x y -> x) "a" 'b'
runPair  pair  (\x y -> y) = (\x y -> y) "a" 'b'

runPair makePair "a" 'b' (\x y -> x)
(\x y -> x) "a" 'b'
 "a"

runPair makePair "a" 'b' (\x y -> y)
(\x y -> y) "a" 'b'
 'b'
```

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Pair type example (6)

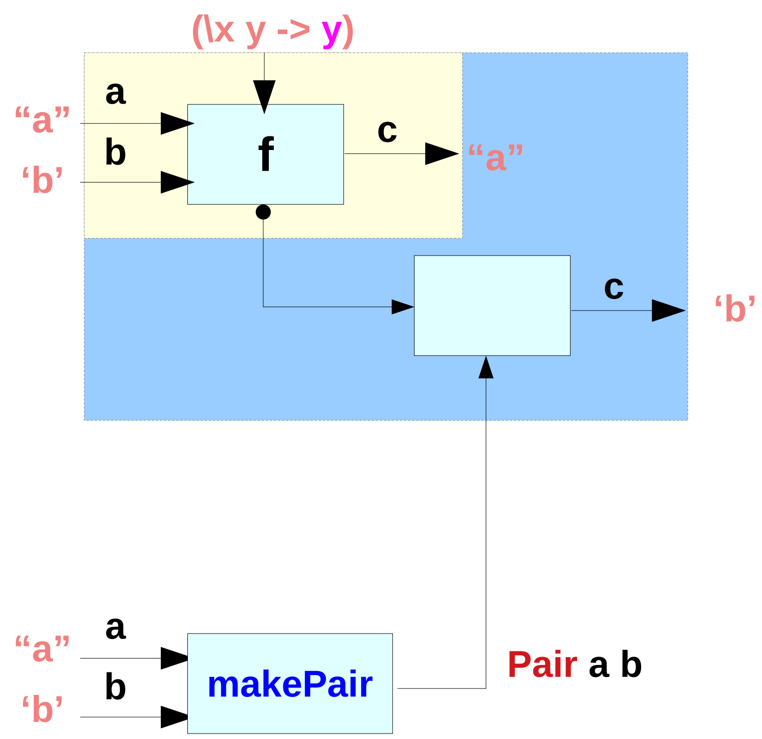
Pair $ \f -> f a b :: Pair a b



pair (\x y -> x)

makePair "a" 'b' (\x y -> x)

Pair $ \f -> f a b :: Pair a b



pair (\x y -> y)

makePair "a" 'b' (\x y -> y)

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf

[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf

Existentials

Existential types, or '**existentials**' for short, provide a way of 'squashing' a group of types into one, single type.

Existentials are part of GHC's type system **extensions**.
They aren't part of Haskell98, and as such you'll have

to either compile any code that contains them
with an extra command-line parameter of

-XExistentialQuantification,

or put at the top of your sources that use existentials.

{-# LANGUAGE ExistentialQuantification #-}

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types