Correlation

Image source: http://commons.wikimedia.org/wiki/File:Gnome-power-statistics.svg, GPL

Lecture 4

Survey Research & Design in Psychology

James Neill, 2018

Readings

Howitt & Cramer (2014)

- Ch 7: Relationships between two or more variables: Diagrams and tables
- Ch 8: Correlation coefficients: Pearson correlation and Spearman's rho
- Ch 11: Statistical significance for the correlation coefficient: A practical introduction to statistical inference
- Ch 15: Chi-square: Differences between samples of frequency data
- Note: Howitt and Cramer doesn't cover point bi-serial correlation

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Overview

1 Covariation
2 Purpose of correlation
3 Linear correlation
4 Types of correlation
5 Interpreting correlation
6 Assumptions / limitations

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Covariation

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e.g., pollen and bees

e.g., study and grades

e.g., nutrients and growth

The world is made of co-variations

e.g., depictions of violence in the environment.

Measure observations and analyse their co-occurrence

e.g., psychological states such as stress and depression.

Covariations are the basis of more complex models

Purpose of correlation

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Purpose of correlation

The underlying purpose of correlation is to help address the question:

What is the
- **relationship** or
- **association** or
- **shared variance** or
- **co-relation**

between **two variables**?

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Purpose of correlation

Other ways of expressing the underlying correlational question include:

To what extent do variables
- **covary**?
- **depend** on one another?
- **explain** one another?

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Linear correlation

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Linear correlation

Extent to which two variables have a simple **linear** (straight-line) relationship.

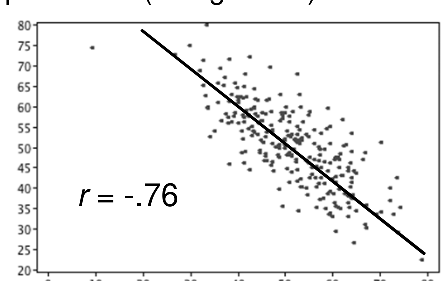

Image source: http://commons.wikimedia.org/wiki/File:Scatterplot_r%3D-.76.png, Public domain

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Linear correlation

The linear relation between two variables is indicated by a correlation's:

- **Direction:** Sign (+ / -) indicates direction of relationship (+ve or -ve slope)
- **Strength:** Size indicates strength (values closer to -1 or +1 indicate greater strength)
- **Statistical significance:** *p* indicates likelihood that the observed relationship could have occurred by chance

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Types of correlation

- No relationship (r ~ 0) (X and Y are independent)
- Linear relationship (X and Y are dependent)
  - As X ↑s, so does Y ($r > 0$)
  - As X ↑s, Y ↓s ($r < 0$)
- Non-linear relationship

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Types of correlation

There are many different measures of correlation.

To decide which type of correlation to use, consider the **levels of measurement** for each variable.

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Types of correlation

- Nominal by nominal: Phi (Φ) / Cramer's *V,* Chi-square
- Ordinal by ordinal: Spearman's rank / Kendall's Tau *b*
- Dichotomous by interval/ratio: Point bi-serial r_{pb}
- Interval/ratio by interval/ratio: Product-moment or Pearson's *r*

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Types of correlation and LOM

	Nominal	Ordinal	Int/Ratio
Nominal	Clustered bar-chart Chi-square, Phi (φ) or Cramer's *V*	⇐ Recode	Clustered bar chart or scatterplot Point bi-serial correlation (r_{pb})
Ordinal		Clustered bar chart or scatterplot Spearman's Rho or Kendall's Tau	⇐⇑Recode
Interval/Ratio			Scatterplot Product-moment correlation (r)

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Nominal by nominal

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Nominal by nominal correlational approaches

- Contingency (or cross-tab) tables
  - Observed frequencies
  - Expected frequencies
  - Row and/or column %s
  - Marginal totals
- Clustered bar chart
- Chi-square
- Phi (ϕ) / Cramer's V

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Contingency tables

- Bivariate frequency tables
- Marginal totals (blue)
- Observed cell frequencies (red)

	Diseased	Disease Free	
Exposed	a	b	n_1
Not Exposed	c	d	n_0
	m_1	m_0	n

Contingency table: Example

Snoring Do you snore? * Smokingr Smoking status Crosstabulation

Count

		Smokingr Smoking status		
		0 Non-smoker	1 Smoker	Total
Snoring Do you snore?	0 yes	50	16	66
	1 no	111	11	122
Total		161	27	188

BLUE = Marginal totals
RED = Cell frequencies

Contingency table: Example

χ^2 = sum of ((observed – expected)^2 / expected)

Snoring Do you snore? * Smokingr Smoking status Crosstabulation

			Smokingr Smoking status		
			0 Non-smoker	1 Smoker	Total
Snoring Do you snore?	0 yes	Count	50	16	66
		Expected Count	56.5	9.5	66.0
	1 no	Count	111	11	122
		Expected Count	104.5	17.5	122.0
Total		Count	161	27	188
		Expected Count	161.0	27.0	188.0

- Expected counts are the cell frequencies that should occur if the variables are not correlated.
- Chi-square is based on the squared differences between the actual and expected cell counts.

Cell percentages

Row and/or column cell percentages can also be useful e.g., ~60% of smokers snore, whereas only ~30%d of non-smokers snore.

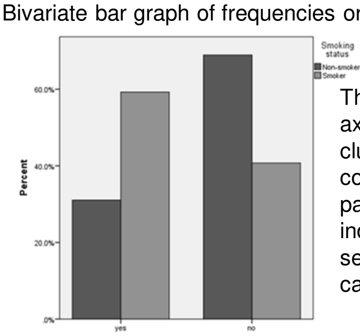
Snoring Do you snore? * Smokingr Smoking status Crosstabulation

% within Smokingr Smoking status

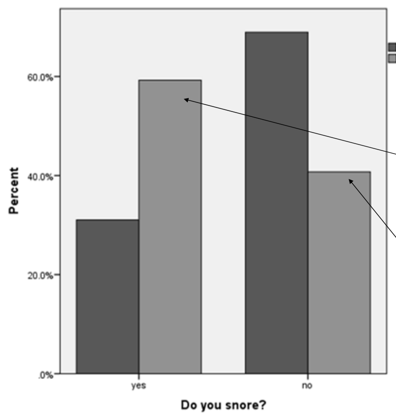
		Smokingr Smoking status		
		0 Non-smoker	1 Smoker	Total
Snoring Do you snore?	0 yes	31.1%	59.3%	35.1%
	1 no	68.9%	40.7%	64.9%
Total		100.0%	100.0%	100.0%

Clustered bar graph

Bivariate bar graph of frequencies or percentages.



The category axis bars are clustered (by colour or fill pattern) to indicate the second variable's categories.



Pearson chi-square test

The value of the test-statistic is

$$X^2 = \sum \frac{(O-E)^2}{E},$$

where

X^2 = the test statistic that approaches a χ^2 distribution.

O = frequencies observed;

E = frequencies expected (asserted by the null hypothesis).

Pearson chi-square test: Example

Smoking (2) x Snoring (2)

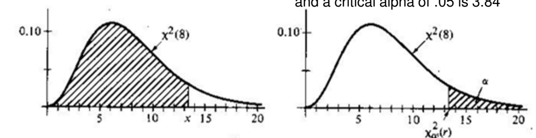
	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	8.073[a]	1	.004		
Continuity Correction[b]	6.883	1	.009		
Likelihood Ratio	7.694	1	.006		
Fisher's Exact Test				.008	.005
Linear-by-Linear Association	8.030	1	.005		
N of Valid Cases	188				

Write-up: χ^2 (1, 188) = 8.07, p = .004

Chi-square distribution: Example

The Chi-Square Distribution

The critical value for chi-square with 1 *df* and a critical alpha of .05 is 3.84



$$P(X \le x) = \int_0^x \frac{1}{\Gamma(r/2)2^{r/2}} w^{r/2-1} e^{-w/2}\, dw$$

	$P(X \le x)$							
	0.010	0.025	0.050	0.100	0.900	0.950	0.975	0.990
r	$\chi^2_{0.99}(r)$	$\chi^2_{0.975}(r)$	$\chi^2_{0.95}(r)$	$\chi^2_{0.90}(r)$	$\chi^2_{0.10}(r)$	$\chi^2_{0.05}(r)$	$\chi^2_{0.025}(r)$	$\chi^2_{0.01}(r)$
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.34
4	0.297	0.484	0.711	1.064	7.779	9.488	11.14	13.28
5	0.554	0.831	1.145	1.610	9.236	11.07	12.83	15.09

Phi (ϕ) & Cramer's *V*

(non-parametric measures of correlation)

Phi (ϕ)

- Use for 2 x 2, 2 x 3, 3 x 2 analyses
  e.g., Gender (2) & Pass/Fail (2)

Cramer's *V*

- Use for 3 x 3 or greater analyses
  e.g., Favourite Season (4) x Favourite Sense (5)

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Phi (ϕ) & Cramer's *V*: Example

Symmetric Measures

		Value	Approximate Significance
Nominal by Nominal	Phi	-.207	.004
	Cramer's V	.207	.004
N of Valid Cases		188	

χ^2 (1, 188) = 8.07, p = .004, **ϕ** = .21

Note that the sign is ignored here (because nominal coding is arbitrary, so the researcher should explain the direction of the relationship)

Ordinal by ordinal

Ordinal by ordinal correlational approaches

- Spearman's rho (r_s)
- Kendall tau (τ)
- Alternatively, use nominal by nominal techniques
  (i.e., recode the variables or treat them as having a lower level of measurement)

Graphing ordinal by ordinal data

- Ordinal by ordinal data is difficult to visualise because it is non-parametric, with many points.
- Consider using:
  - Non-parametric approaches (e.g., clustered bar chart)
  - Parametric approaches (e.g., scatterplot with line of best fit)

Spearman's rho (r_s) or Spearman's rank order correlation

- For ranked (ordinal) data
  - e.g., Olympic Placing correlated with World Ranking
- Uses product-moment correlation formula
- Interpretation is adjusted to consider the underlying ranked scales

Kendall's Tau (τ)

- Tau a
  - Does not take joint ranks into account
- Tau b
  - Takes joint ranks into account
  - For square tables
- Tau c
  - Takes joint ranks into account
  - For rectangular tables

Ordinal correlation example

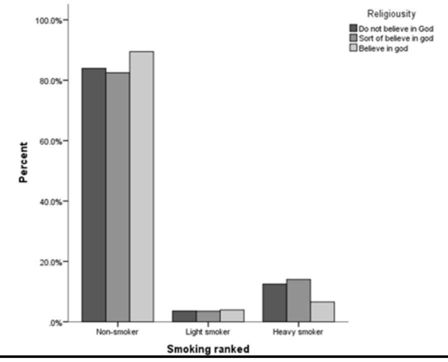
Godranked Religiousity

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0 Do not believe in God	56	29.5	29.5	29.5
	1 Sort of believe in god	57	30.0	30.0	59.5
	2 Believe in god	77	40.5	40.5	100.0
	Total	190	100.0	100.0	

Smokingranked Smoking ranked

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0 Non-smoker	162	85.3	85.7	85.7
	1 Light smoker	7	3.7	3.7	89.4
	2 Heavy smoker	20	10.5	10.6	100.0
	Total	189	99.5	100.0	
Missing	System	1	.5		
Total		190	100.0		

Ordinal correlation example



Ordinal correlation example

Correlations

			Godranked Religiousity	Smokingranked Smoking ranked
Kendall's tau_b	Godranked Religiousity	Correlation Coefficient	1.000	-.071
		Sig. (2-tailed)	.	.298
		N	190	189
	Smokingranked Smoking ranked	Correlation Coefficient	-.071	1.000
		Sig. (2-tailed)	.298	.
		N	189	189

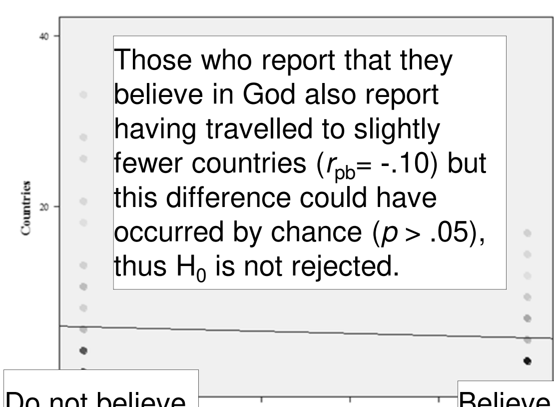
τ_b = -.07, p = .298

There is a small effect towards non-smokers being more likely to believe in God, but this could have occurred through sampling error.

Dichotomous by interval/ratio

Point-biserial correlation (r_{pb})

- One dichotomous & one interval/ratio variable
  - e.g., belief in god (yes/no) and number of countries visited
- Calculate as for Pearson's product-moment *r*
- Adjust interpretation to consider the direction of the dichotomous scales



Those who report that they believe in God also report having travelled to slightly fewer countries (r_{pb}= -.10) but this difference could have occurred by chance (p > .05), thus H_0 is not rejected.

Point-biserial correlation (r_{pb}): Example

Correlations

			b4r God	b8 Countries
b4r God	0 = No 1 = Yes	Pearson Correlation	1	-.095
		Sig. (2-tailed)		.288
		N	127	127
b8 Countries		Pearson Correlation	-.095	1
		Sig. (2-tailed)	.288	
		N	127	190

Interval/ratio by interval/ratio

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Scatterplot

- Plot each pair of observations (X, Y)
  - x = predictor variable (independent; IV)
  - y = criterion variable (dependent; DV)
- By convention:
  - IV on the x (horizontal) axis
  - DV on the y (vertical) axis
- Direction of relationship:
  - +ve = trend from bottom left to top right
  - -ve = trend from top left to bottom right

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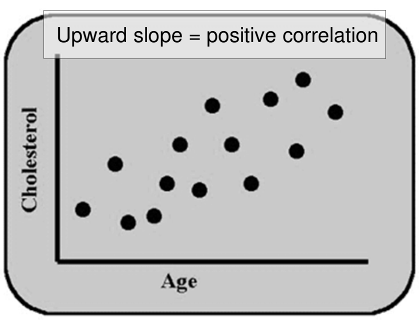
Scatterplot showing relationship between age & cholesterol with line of best fit



Line of best fit

- The correlation between 2 variables is a measure of the degree to which pairs of numbers (points) cluster together around a best-fitting straight line
- Line of best fit: y = a + bx
- Check for:
  - outliers
  - linearity

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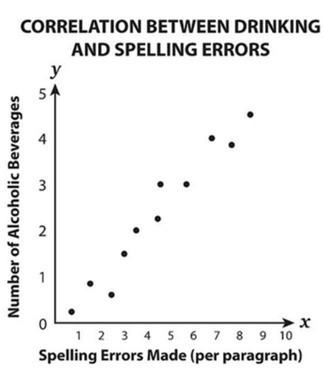
What's wrong with this scatterplot?

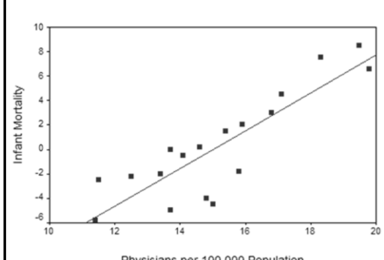
Y-axis should be DV (outcome)

X-axis should be IV (predictor)



Scatterplot example:

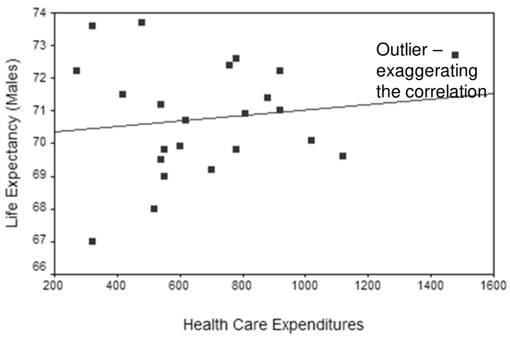
Strong positive (.81)



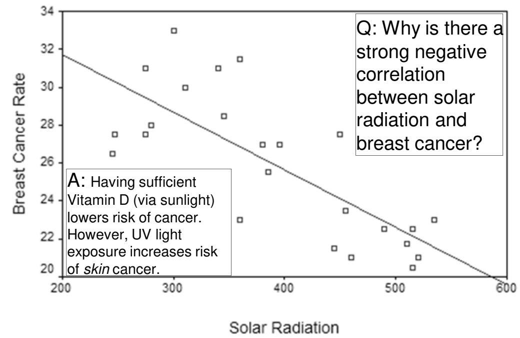
Q: Why is infant mortality positively linearly associated with the number of physicians (with the effects of GDP removed)?

A: Because more doctors tend to be deployed to areas with infant mortality (socio-economic status aside).

Scatterplot example:
Weak positive (.14)



Scatterplot example:
Moderately strong negative (-.76)



Pearson product-moment correlation (*r*)

- The product-moment correlation is the **standardised covariance**.

$$r_{X,Y} = \frac{\text{cov}(X,Y)}{S_X S_Y}$$

Covariance

- Variance shared by 2 variables

$$Cov_{XY} = \frac{\Sigma(X-\overline{X})(Y-\overline{Y})}{N-1}$$

← Cross products
← *N* - 1 for the samp

- Covariance reflects the direction of the relationship:
  - +ve cov indicates +ve relationship
  - -ve cov indicates -ve relationship
- Covariance is unstandardised.

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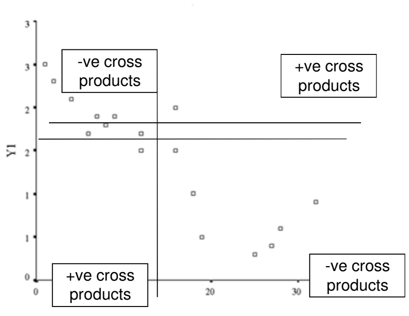
Covariance: Cross-products



Covariance → Correlation

- Size depends on the measurement scale → Can't compare covariance across different scales of measurement (e.g., age by weight in kilos <u>versus</u> age by weight in grams).
- Therefore, **standardise** covariance (divide by the cross-product of the SDs) → correlation
- Correlation is an effect size - i.e., standardised measure of strength of linear relationship

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Example quiz question: Covariance, *SD*, and correlation

The covariance between *X* and *Y* is 1.2. The *SD* of *X* is 2 and the *SD* of *Y* is 3. The correlation is:

a. 0.2
b. 0.3
c. 0.4
d. 1.2

$$r_{X,Y} = \frac{\mathrm{cov}(X, Y)}{S_X S_Y}$$

Answer:
1.2 / 2 x 3 = 0.2

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Hypothesis testing

Almost all correlations are not 0. So, hypothesis testing seeks to answer:

- What is the **likelihood** that an observed relationship between two variables is “true” or “real”?
- What is the **likelihood** that an observed relationship is simply due to chance (sampling error)?

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Significance of correlation

- **Null hypothesis (H_0)**: **rho** ρ = 0
  i.e., no “true” relationship in the population
- **Alternative hypothesis (H_1)**: ρ •• 0
  i.e., there is a real relationship in the population
- Initially, assume $\mathbf{H_0}$ is true, and then evaluate whether the data support $\mathbf{H_1}$.
- **ρ (rho)** = *population* product-moment correlation coefficient

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How to test the null hypothesis

- Select a critical value (alpha (α)); commonly .05
- Use a 1- or 2-tailed test; 1-tailed if hypothesis is directional
- Calculate correlation and its *p* value. Compare to the critical alpha value.
- If $p <$ critical alpha, correlation is statistically significant, i.e., there is less than critical alpha chance that the observed relationship is due to random sampling variability.

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Correlation - SPSS output

Correlations

		Cigarette Consumption per Adult per Day	CHD Mortality per 10,000
Cigarette Consumption per Adult per Day	Pearson Correlation		
	Sig. (2-tailed)		
	N		
CHD Mortality per 10,000	Pearson Correlation	.713*	
	Sig. (2-tailed)	.000	
	N	21	

**. Correlation is significant at the 0.01 level (2-tailed).

Errors in hypothesis thesting

- **Type I error**:
  decision to reject $\mathbf{H_0}$ when H_0 is true
- **Type II error**:
  decision to not reject $\mathbf{H_0}$ when H_0 is false
- A significance test outcome depends on the statistical power which is a function of:
  - Effect size (*r*)
  - Sample size (*N*)
  - Critical alpha level (α_{crit})

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Significance of correlation

df (*N* - 2)	critical p = .05
5	.67
10	.50
15	.41
20	.36
25	.32
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The higher the *N*, the smaller the correlation required for a statistically significant result – why?

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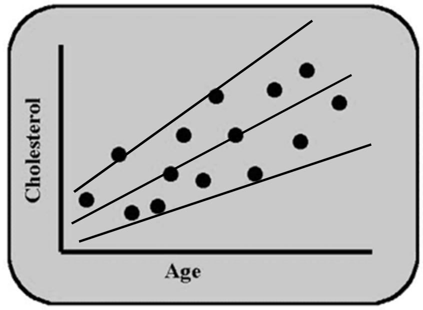
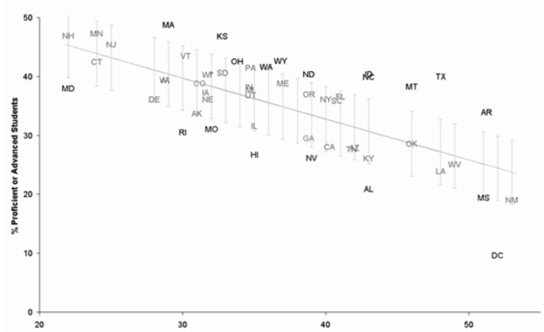
Scatterplot showing a confidence interval for a line of best fit





Scatterplot showing a confidence interval for a line of best fit

Image source: http://www.chriscorrea.com/2005/poverty-and-performance-on-the-naep/

Practice quiz question:
Significance of correlation

If the correlation between Age and Performance is statistically significant, it means that:

a. there is an important relationship between the variables
b. the true correlation between the variables in the population is equal to 0
c. the true correlation between the variables in the population is not equal to 0
d. getting older causes you to do poorly on tests

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Interpreting correlation

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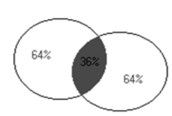
Coefficient of Determination (r^2)

- CoD = The proportion of variance in one variable that can be accounted for by another variable.
- e.g., $r = .60$, $r^2 = .36$ or 36% of shared variance



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Interpreting correlation
(Cohen, 1988)

- A correlation is an **effect size**
- Rule of thumb:

Strength	r	r^2
Weak:	.1 - .3	1 - 9%
Moderate:	.3 - .5	10 - 25%
Strong:	>.5	> 25%

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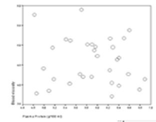
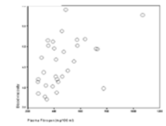
Size of correlation
(Cohen, 1988)

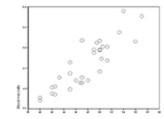
WEAK (.1 - .3)



MODERATE (.3 - .5)



STRONG (> .5)



Interpreting correlation
(Evans, 1996)

Strength	r	r^2
very weak	.00 - .19	(0 to 4%)
weak	.20 - .39	(4 to 16%)
moderate	.40 - .59	(16 to 36%)
strong	.60 - .79	(36% to 64%)
very strong	.80 - 1.00	(64% to 100%)

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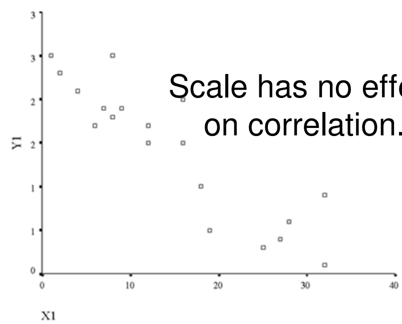
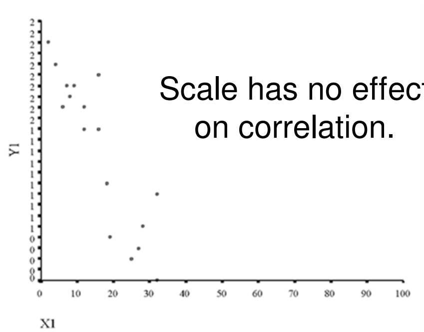
Correlation of this scatterplot = -.9



Scale has no effect on correlation.

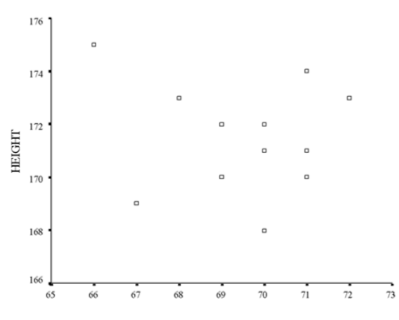
Correlation of this scatterplot = -.9



Scale has no effect on correlation.

What is the correlation of this scatterplot?

a. -.5
b. -1
c. 0
d. .5
e. 1



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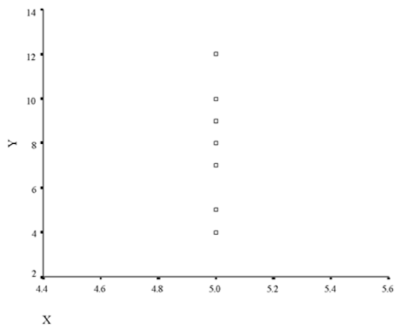

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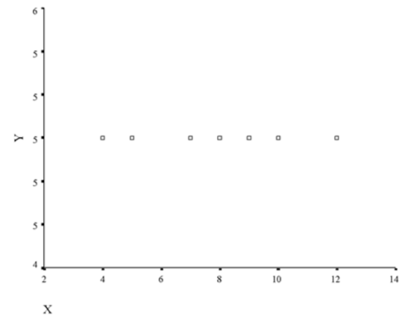
e. 1



Write-up: Example

"Number of children and marital satisfaction were inversely related (*r* (48) = -.35, *p* < .05), such that contentment in marriage tended to be lower for couples with more children. Number of children explained approximately 10% of the variance in marital satisfaction, a small-moderate effect."

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Assumptions and limitations

(Pearson product-moment linear correlation)

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Assumptions and limitations

1 Levels of measurement
2 Normality
3 Linearity
  1 Effects of outliers
  2 Non-linearity
4 Homoscedasticity
5 No range restriction
6 Homogenous samples
7 Correlation is not causation
8 Dealing with multiple correlations

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Normality

- X and Y data should be sampled from populations with normal distributions
- Do not overly rely on any single indicator of normality; use histograms, skewness and kurtosis (e.g., within -1 and +1)
- Inferential tests of normality (e.g., Shapiro-Wilks) are overly sensitive when sample is large

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Effects of outliers

- Outliers can disproportionately increase or decrease *r*.
- Options
  - compute *r* with & without outliers
  - get more data for outlying values
  - recode outliers as having more conservative scores
  - transformation
  - recode variable into lower level of measurement and a non-parametric approach

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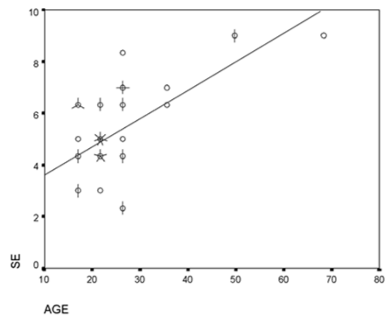
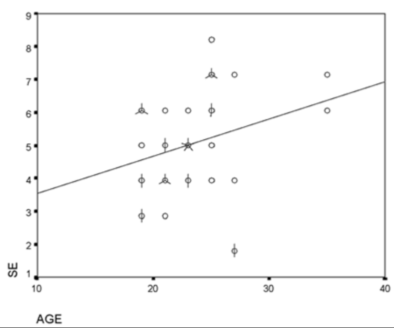
Age and self-esteem

(r = .63)



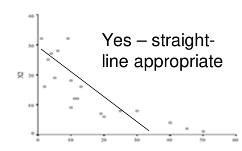
Age and self-esteem

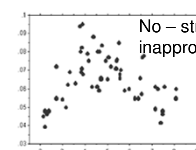
(outliers removed) r = .23



Non-linear relationships

Check scatterplot
Can a linear relationship "capture" the lion's share of the variance?
If so,use *r*.





Non-linear relationships

If non-linear, consider:

- Does a linear relation help?
- Use a non-linear mathematical function to describe the relationship between the variables
- Transforming variables to "create" linear relationship

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Scedasticity

- **<u>Homo</u>**scedasticity refers to even spread of observations about a line of best fit
- **<u>Hetero</u>**scedasticity refers to uneven spread of observations about a line of best fit
- Assess visually and with Levene's test

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Scedasticity

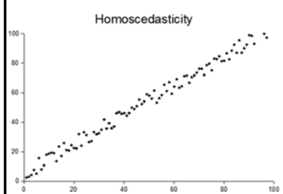


Image source:
https://commons.wikimedia.org/wiki/File:Homoscedasticity.png

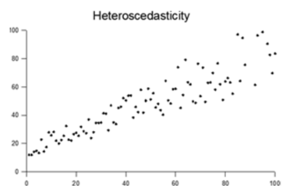
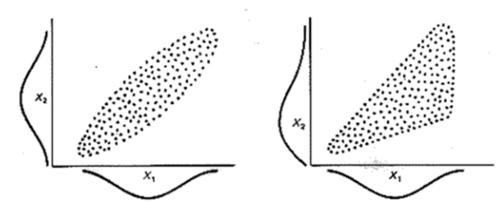


Image source:
https://commons.wikimedia.org/wiki/File:Heteroscedasticity.png

Scedasticity



Homoscedasticity with both variables normally distributed

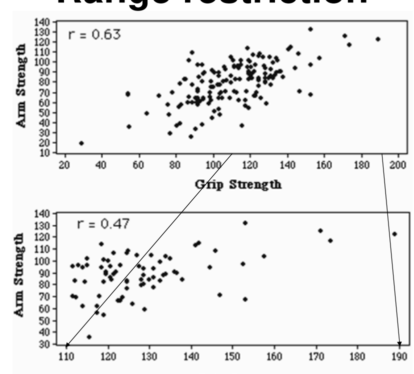
Heteroscedasticity with skewness on one variable

Image source: Unknown

Range restriction

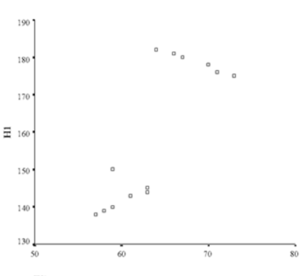
- Range restriction is when the sample contains a restricted (or truncated) range of scores
  - e.g., level of hormone X and age < 18 might have linear relationship
- If range is restricted, be cautious about generalising beyond the range for which data is available
  - e.g., level of hormone X may not continue to increase linearly with age after age 18

Range restriction



Heterogeneous samples

- Sub-samples (e.g., males & females) may artificially increase or decrease overall *r*.
- Solution - calculate *r* separately for sub-samples & overall; look for differences

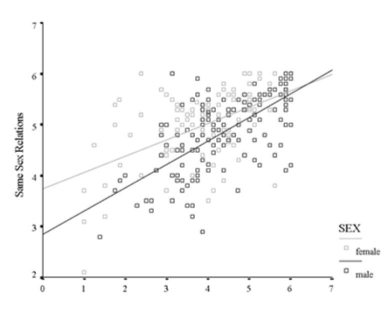


Heterogeneous samples

Same-sex & Opposite-sex Relations by Gender

♂ $r = .67$
$r^2 = .45$

♀ $r = .52$
$r^2 = .27$

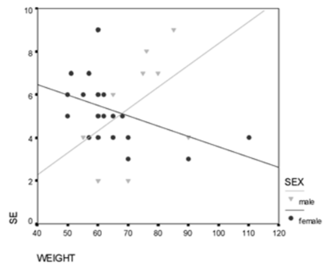


Heterogeneous samples

Weight and Self-esteem by Gender

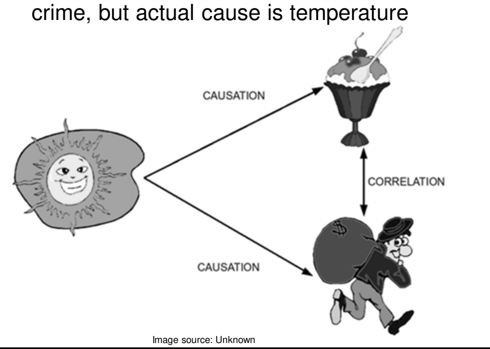
♂ *r* = .50

♀ *r* = -.48



Correlation is not causation

e.g.,correlation between ice cream consumption and crime, but actual cause is temperature



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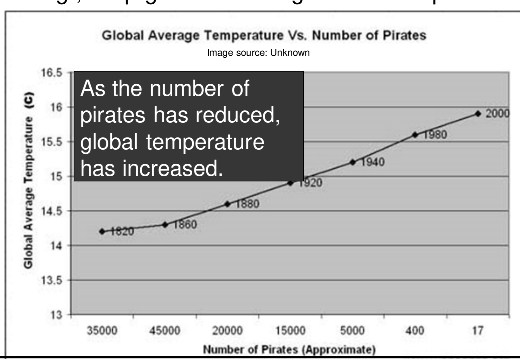
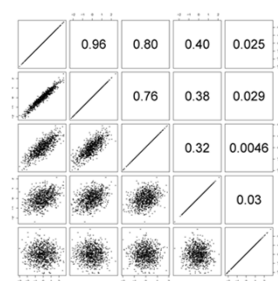
Correlation is not causation

e.g., Stop global warming: Become a pirate



Dealing with several correlations

Scatterplot matrices organise scatterplots and correlations amongst several variables at once. However, they are not sufficiently detailed for more than about five variables at a time.



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Correlation matrix

Example of an APA Style Correlation Table

Table 1.

Correlations Between Five Life Effectiveness Factors for Adolescents and Adults (N = 3640)

	Time Manage-ment	Social Compet-ence	Achieve-ment Motivation	Intellectual Flexibility	Task Leadership
Time Management		.36	.53	.31	.42
Social Competence			.37	.32	.57
Achievement Motivation				.42	.41
Intellectual Flexibility					.37
Task Leadership					

Summary

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Summary: Correlation

1 The world is made of covariations.
2 Covariations are the building blocks of more complex multivariate relationships.
3 Correlation is a standardised measure of the covariance (extent to which two phenomenon co-relate).
4 Correlation does not prove causation - may be opposite causality, bi-directional, or due to other variables.

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Summary: Purpose of correlation

The underlying purpose of correlation is to help address the question:

What is the
- **relationship** or
- **association** or
- **shared variance** or
- **co-relation**

between **two variables**?

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Summary: Types of correlation

- Nominal by nominal:
  Phi (Φ) / Cramer's *V,* Chi-square
- Ordinal by ordinal:
  Spearman's rank / Kendall's Tau *b*
- Dichotomous by interval/ratio:
  Point bi-serial r_{pb}
- Interval/ratio by interval/ratio:
  Product-moment or Pearson's *r*

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Summary: Correlation steps

1 Choose correlation and graph type based on levels of measurement.
2 Check graphs (e.g., scatterplot):
  – Linear or non-linear?
  – Outliers?
  – Homoscedasticity?
  – Range restriction?
  – Sub-samples to consider?

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Summary: Correlation steps

3 Consider
  – Effect size (e.g., Φ, Cramer's *V*, *r*, r^2)
  – Direction
  – Inferential test (*p*)
4 Interpret/Discuss
  – Relate back to hypothesis
  – Size, direction, significance
  – Limitations e.g.,
    - Heterogeneity (sub-samples)
    - Range restriction
    - Causality?

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Summary: Interpreting correlation

- Coefficient of determination
  – Correlation squared
  – Indicates % of shared variance

Strength	***r***	r^2
Weak:	.1 - .3	1 – 10%
Moderate:	.3 - .5	10 - 25%
Strong:	> .5	> 25%

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Summary: Assumptions & limitations

1 Levels of measurement
2 Normality
3 Linearity
   1 Effects of outliers
   2 Non-linearity
4 Homoscedasticity
5 No range restriction
6 Homogenous samples
7 Correlation is not causation

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References

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Next lecture

Exploratory factor analysis

- Introduction to factor analysis
- Exploratory factor analysis examples
- EFA steps / process

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