What we can learn from E0 transitions and how?

T. Kibèdi (ANU)
The Absolute Intensities and Internal Conversion Coefficients of the $\gamma$-Radiations of Radium B and Radium C.


(Received July 1, 1930.)

Highly converted K, L1, M1 lines corresponding to 1.426 MeV

1.4155 MeV E0 in $^{214}$Po

$T_{1/2}=99(3)$ ps; $\rho^2=0.0013(2)$

0 → 0 transition: quantum transition forbidden
**Transition probability**

\[ W_T = W_\gamma + W_{CE} + W_\pi \]

**Conversion Coefficient**

\[ \alpha_{ce,\pi} = \frac{W_{ce,\pi}}{W_\gamma} \]

**CE & \pi transitions**

\[ W_{ce,\pi} = W_\gamma \times \alpha_{ce,\pi} \]

**EO transitions**

\[ W_{ce,\pi} = \rho^2 (0^+ \rightarrow 0^+) \times \Omega_{ce,\pi}(Z,K) \]

**Monopole strength parameter**

\[ \rho = \frac{\langle 0^+_f \mid \sum e_j r_j^2 \mid 0^+_i \rangle}{eR^2} = \frac{\langle 0^+_f \mid m(E0) \mid 0^+_i \rangle}{eR^2} \]

**Reduced transition probability**

\[ B(E0) = e^2 R^4 \rho^2 \]
$E_0$ transitions

**Transition probability**

$$W_T = W_\gamma + W_{CE} + W_\pi$$

**Conversion Coefficient**

$$\alpha_{ce,\pi} = \frac{W_{ce,\pi}}{W_\gamma}$$

**CE & $\pi$ transitions**

$$W_{ce,\pi} = \frac{W_\gamma}{\alpha_{ce,\pi}}$$

**EO transitions**

$$W_{ce,\pi} = \rho^2(0^+ \rightarrow 0^+) \times \Omega_{ce,\pi}(Z,\kappa)$$

- L$\neq$0: $W_\gamma$ and $W_{ce,\pi}$
- L=0 i.e. $E_0$: only $W_{ce,\pi}$!

Need to measure electrons and electron-positron pairs

What can we measure?
\[ W_{ce,\pi} = \rho^2(0^+ \rightarrow 0^+) \times \Omega_{ce,\pi}(Z,\kappa) \]

\[ \Omega_{ce,\pi}(Z,\kappa) \] - E0 electronic factor, defined by Church & J. Weneser (1956)

- CE predominantly in \( s_{1/2} \) and less on \( p_{1/2} \) shells
- K/L weak dependence on Z (4 to 9)
- Pair conversion dominant at low Z
- Double photon emission; 2E1/2M1 (mixed or pure); 10^{-4} decay branch
Measuring E0 transitions

- Talks by John Wood
- Talks by M. Reed, T. Eriksen, L. Evitts

Measuring even the strongest E0 ($10^3 \rho^2(E0)=500$) is often difficult! HIAS talk on Thursday

Physics opportunities
Physics opportunities
Talks by John Wood, Hiroshi Watanabe, Eiji Ideguchi, Lee Evitts, Paul Garrett, Mitch Allmond, Robert Page, Sean Liddick, Wolfram Korten

CE / pair spectrometer
talks by Rob Bark, James Smallcombe, Hiroshi Watanabe, Tomas Eriksen, Matt Reed, Greg Lane, Pete Jones

Absolute transition rates
Talk by Steve Yates

We also need
Conversion coefficients and E0 electronic factors
E2/M1 mixing ratios (for J→J; J≠0)
Experiment: 182 ICC's; uncertainty ≤5%
Theory: “Frozen orbital” approximation
Band et al. ADNDT 81 (2002) 1; adopted for BrIcc

Down to the last percent!

Talks by Boon Lee and Aqeel Akber
\[ \Omega(k_o) = 2\pi (2j + 1) \left( \frac{e\gamma_o}{2|k_o|(2|k_o| + 1)} \right)^2 = 8\pi\alpha\omega A(E0) \]

\[ \Omega(\pi) = \frac{1}{(1 + \gamma)^2} \int_1^{E_\pi-1} p_+p_- (E_+E_- - \gamma^2) F(Z, E+) F(Z, E-) dE_+ \]


D.A. Bell et al., Can. J. Phys. 48 (1970) 2542: \( \Omega(k_o) \) for K, L1, L2; Z=40-102

A. Passoja & T. Salonen, JYFL RR-2/86 (1986): \( \Omega(k_o, \pi) \) for K; Z=8-40

K-shell satisfactory agreement; L1 and L2, K/L up to 10-15% differences; NO data for M and higher shells

**New calculations**

\( \Omega(k_o) \): using H.C. Pauli, and U. Raff, Comp. Phys. Comm. 9 392 (1975)
Relativistic Hartree-Fock-Slater, for K, L1, L2, M1, M2, ...; Z=2-96; 1-6000 keV
In collaboration width G. Gosselin, V. Meot, M. Pascal (CEA, France)

\( \Omega(\pi) \) & \( d\Omega(\pi)/dE_+ d\theta \): Using Wilkinson`s model, Effect of Coulomb field is taken into account; same as Passoja, Tomas Eriksen`s PhD
$\Omega(E0)$ - how good are they?

- $\Omega_{\kappa,\pi}(E0) \sim I_{\kappa,\pi}(E0)$

- Only ratios of $\Omega_{\kappa,\pi}(E0)$ could be measured
  - $K/\pi$
  - $K/L, K/LM, K/MN$
  - $L/M$

- Theory: new ANU & CEA (France) calculations

- Experiment vs. theory: Theory overestimate experiment by $\sim 6\%$

Experimental E0 sub-shell ratios are needed
\( \rho^2(E0) \) from experiments - \( J=0 \)

Simplest case: \(^4\text{He}, \, ^{14}\text{C}, \, ^{16}\text{O}, \, ^{40}\text{Ca}, \, ^{68}\text{Ni}, \, ^{72}\text{Ge}, \, ^{72}\text{Kr}, \, ^{96}\text{Zr}, \, ^{98}\text{Mo}, \, ^{190,192,194}\text{Pb} \) (E2 energetically forbidden)

\[
\rho^2(E0) = \frac{1}{[\Omega_{ce}(E0) + \Omega_\pi(E0)] \times \tau(E0)}
\]

\(^{96}\text{Zr}: E^*=1581.6 \text{ keV}; T_{1/2}=38.0(7) \text{ ns}; 1997\text{Bh08} \)

\[
\Omega_K + \Omega_{L1} + \Omega_{L2} + \Omega_\pi = 2.41\times10^9; \, K/\pi=5.4
\]

\[10^3 \rho^2(E0) = 7.57(14)\]

New tabulations: \( \Omega_{CE} = 2.07\times10^9; \, \Omega_\pi = 3.59\times10^8; \, K/\pi=5.1\)
\[ \rho^2(E0) = \frac{I_{ce,\pi}(E0)}{I_{ce,\pi}(E2)} \times \frac{\alpha_{ce,\pi}(E2)}{\Omega_{ce,\pi}(E0)} \times W_\gamma(E2) \]

Typical case; E2 allowed; \( W_\gamma(E2) \) is known

\[ \frac{\rho^2(E0)}{E0} = 3.7(6) \times 10^3 \]

**118Sn**: \( E^* = 1758.3 \text{ keV}; T_{1/2} = 21(3) \text{ ps}; E0 = 1758.1; E2 = 528.7 \)

\[ I_K(E0)/I_K(E2) = 0.32(6) \quad 1981Ba05 \]
\[ I_K(E0)/I_K(E2) = 0.22(2) \quad 1982Ka09 \]

\[ \Omega_K + \Omega_{L1} + \Omega_{L2} = 1.24 \times 10^{10} \]

\[ X(E0/E2) = 0.0070(6) \]

New tabulations: \( \Omega_{CE} = 1.23 \times 10^{10}; \Omega_\pi = 1.68 \times 10^9; K/\pi = 7.3 \)
**E0+E2+M1 conversion coefficient**

\[
\alpha_{ce,\pi}(E0 / E2 / M1) = \frac{I_{ce,\pi}(E0) + I_{ce,\pi}(E2) + I_{ce,\pi}(M1)}{I_\gamma(E2) + I_\gamma(M1)}
\]

**E0/E2 mixing ratio** (Church, Rose & Weneser 1958)

\[
q_{ce,\pi}^2(E0 / E2) = \frac{I_{ce,\pi}(E0)}{I_{ce,\pi}(E2)}
\]

**NOTE**

\[
q_\pi^2 = \frac{I_{\pi}(E0)}{I_{\pi}(E2)} = q_K^2 \times \frac{\Omega_{\pi}(E0)}{\Omega_K(E0)} \times \frac{\alpha_K(E2)}{\alpha_\pi(E2)}
\]

\[
\delta_\gamma^2(E2 / M1) = \frac{W_\gamma(E2)}{W_\gamma(M1)}
\]

\[
\alpha_{ce,\pi}(E0 / E2 / M1) = \frac{\delta_\gamma^2 [1 + q_{ce,\pi}^2] \alpha_{ce,\pi}(E2) + \alpha_{ce,\pi}(M1)}{1 + \delta_\gamma^2}
\]

\[
\rho^2(E0) = q_{ce,\pi}^2(E0 / E2) \times \frac{\alpha_{ce,\pi}(E2)}{\Omega_{ce,\pi}(E0)} \times W_\gamma(E2)
\]
Electron penetrates into the nucleus; increased conversion probability

Highly hindered transition; mostly M1
\[ \alpha(M1)[1 + B_1(M1)\lambda + B_2(M1)\lambda^2] \]

If \( q^2(E0/E2) > 2 \) and \( \delta(E2/M1) > 3 \) penetration effect negligible

\[ \alpha_{ce}(E0 / E2 / M1) = \frac{\delta_Y^2 [1 + q_{ce}^2] \alpha_{ce}(E2) + \alpha_{ce}(M1)}{1 + \delta_Y^2} \]
\[ \rho^2(E0) = q_{ce,\pi}^2(E0 / E2) \times \frac{\alpha_{ce,\pi}(E2)}{\Omega_{ce,\pi}(E0)} \times W_\gamma(E2) \]

158\textsuperscript{Gd} - 1

\begin{align*}
10.6 & \quad 2^+ \quad 3260 \\
0^+ & \quad 1176 \\
116.5 (106) & \\
1246 (12) & \\
1180 (106) & \\
998 (106) & \\
\text{other (128)} & \\
\delta & \quad 8 \times 10^{-8} \text{ (NSD)} \\
\end{align*}

\[ d_K = 0.00278 \quad \text{(Greenwood)} \]
\[ d_{K1} = 0.00268 \]
\[ d_{K2} = 0.00166 \]

\[ I_{K0}^{E0} \leq 0.045 \quad \text{(if } E2 + E0 \text{)} \]
\[ 0.0040 \quad \text{(if } M1 + E0 \text{)} \]

\[ \delta \text{ is assumed} \]

John Wood
E0 rare earth-actinide evaluation for
NP A651 (1999) 323
Collective model prediction on $\delta(E2/M1)$

For $^{158}\text{Gd}(1.18\text{ MeV})$, $\delta_{DF} = 19.4$

\[ \delta_{DF} = 3.56 \times 10^{-3} E_\gamma (\text{MeV}) \times A^{5/3} \]

Davydov & Filippov (1958)
rotational model without change in shape; $2^+ \rightarrow 2^+$

\[ \delta_W = 1.521 \times 10^{-3} E_\gamma (\text{MeV}) \times A^{2/3} \]

"Weisskopf Unit" for $\delta$ from $B_W(E\gamma)$ and $B_W(M\lambda)$

If no absolute transition rates are known

\[ X(E_0/E_2) = \frac{B(E_0)}{B(E_2)} = \frac{e^2 R^4 q^2(E_0)}{B(E_2)} \]

Reduced E0/E2 branching ratio (Rasmussen 1960)

**Experimental value**

\[ X(E_0/E_2) = 2.54 \times 10^9 \times A^3 \times q_{ce,\pi}^2(E_0/E_2) \times \frac{\alpha_{ce,\pi}(E_2)}{\Omega_{ce,\pi}(E_0)} \times E_\gamma^5(E_2) \]
On the boundary of shape co-existence

\[ K^{\pi}=0^+ + K^{\pi}=2^+ \]

\[ E_0 \& E_0+E2+M1 \]

Transitions

On the boundary of shape co-existence

4-bands mixing calculations reproduce the spin-dependence of $X(B(E0)/B(E2))$, but overestimate their value.

What physics we can learn from the $X$-values?

The $\rho(E0)$ strength can be obtained from:

- $B(E0)$ from $(e,e')$
- Lifetime measurement combined with $\gamma$, CE & $\pi$

$\rho(E0)$ single particle unit:

- No single transition in shell model
- Bohr & Mottelson from harmonic oscillator model:

$$\langle m(E0) \rangle_{sp} = 1.0 \ A^{1/3} \ e fm^2$$

$$\rho_{sp} = 0.7 \ A^{-1/3}$$

Wilkinson from

$$\frac{\langle 0_f^+ \mid \sum e_j r_j^2 \mid 0_i^+ \rangle}{eR^2}$$

and to fit exp. on $^{16}O$, $^{40}Ca$, $^{90}Zr$:

$$\langle m(E0) \rangle_{sp} = 0.65 \ A^{2/3} \ e fm^2$$

Need to adopt a unit $X=B(E0)/B(E2)$ from relative $\gamma$, CE & $\pi$ much more accessible.

TK & Spear, ADNDT 89 (2005) 77