DFT Frequency (4A)

- Angular Frequency
- Negative Frequency
- Fundamental Frequency
- Harmonic Frequency
- Sampling Frequency
- Normalized Frequency

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Angular Frequency

Frequency $f = \frac{1}{T}$ (Hz: cycles per second)

1Hz → event repeats once per second

Angular Frequency $\omega = \frac{2\pi}{T}$ (radians per second)

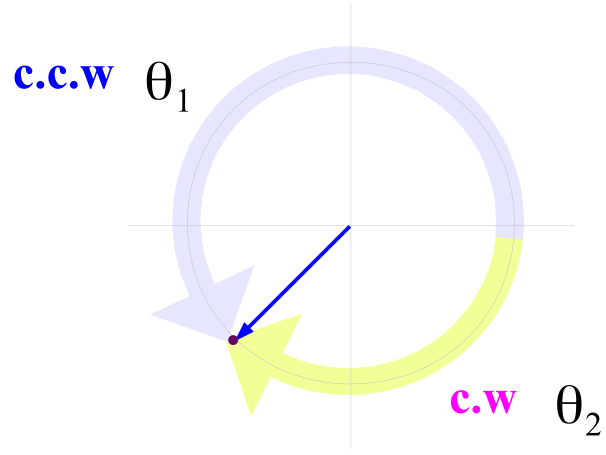
One revolution = 2π radian (speed = $\frac{\text{displacement}}{\text{time}}$)

$$\omega = 2\pi f = 2\pi \frac{1}{T}$$ → **Angular Speed**

$$\theta = \omega t = 2\pi f t$$ → **Phase**

Negative Frequency

An angle can be measured either by a positive angle or a negative angle.



$$e^{+j\omega_1 t} = \cos(+\omega_1 t) + j\sin(+\omega_1 t)$$

$$e^{-j\omega_2 t} = \cos(-\omega_2 t) + j\sin(-\omega_2 t)$$

Angle	**c.c.w (+)**	**Positive Angle**	$\theta_1 = \omega_1 t$	$(\theta_1 > 0)$
	c.w (–)	**Negative Angle**	$\theta_2 = \omega_2 t$	$(\theta_2 < 0)$
Angular Speed	**c.c.w (+)**	**Positive Frequency**	$\omega_1 = +\frac{2\pi}{T}$	$(\omega_1 > 0)$
	c.w (–)	**Negative Frequency**	$\omega_2 = -\frac{2\pi}{T}$	$(\omega_2 < 0)$

DFT Matrix

$$
\begin{bmatrix}
W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\
W_8^0 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^0 & W_8^2 & W_8^4 & W_8^6 \\
W_8^0 & W_8^3 & W_8^6 & W_8^1 & W_8^4 & W_8^7 & W_8^2 & W_8^5 \\
W_8^0 & W_8^4 & W_8^0 & W_8^4 & W_8^0 & W_8^4 & W_8^0 & W_8^4 \\
W_8^0 & W_8^{-3} & W_8^{-6} & W_8^{-1} & W_8^{-4} & W_8^{-7} & W_8^{-2} & W_8^{-5} \\
W_8^0 & W_8^{-2} & W_8^{-4} & W_8^{-6} & W_8^0 & W_8^{-2} & W_8^{-4} & W_8^{-6} \\
W_8^0 & W_8^{-1} & W_8^{-2} & W_8^{-3} & W_8^{-4} & W_8^{-5} & W_8^{-6} & W_8^{-7}
\end{bmatrix}
$$

** still symmetric matrix*

c.w (–)

c.c.w (+)

$$W_N^{nk \pm N} = W_N^{nk}$$

$$W_8^{nk} = e^{-j\left(\frac{2\pi}{8}\right)nk}$$

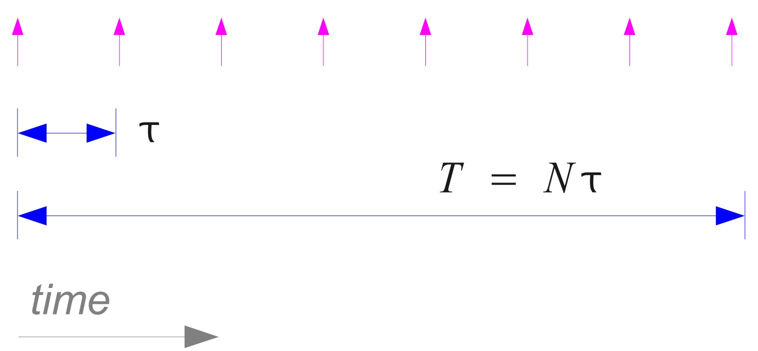
Exponents and Strides

	n=0		n=1		n=2		n=3		n=4		n=5		n=6		n=7
k=0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0
k=1	0	-1	-1	-1	-2	-1	-3	-1	-4	-1	-5	-1	-6	-1	-7
k=2	0	-2	-2	-2	-4	-2	-6	-2	0	-2	-2	-2	-4	-2	-6
k=3	0	-3	-3	-3	-6	-3	-1	-3	-4	-3	-7	-3	-2	-3	-5
k=4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4
k=5	0	+3	+3	+3	+6	+3	+1	+3	+4	+3	+7	+3	+2	+3	+5
k=6	0	+2	+2	+2	+4	+2	+6	+2	0	+2	+2	+2	+4	+2	+6
k=7	0	+1	+1	+1	+2	+1	+3	+1	+4	+1	+5	+1	+6	+1	+7

Fundamental and Harmonic Frequencies

Exponents and Strides of DFT Matrix

	n=0		n=1		n=2		n=3		n=4		n=5		n=6		n=7
k=0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0
k=1	0	-1	-1	-1	-2	-1	-3	-1	-4	-1	-5	-1	-6	-1	-7
k=2	0	-2	-2	-2	-4	-2	-6	-2	0	-2	-2	-2	-4	-2	-6
k=3	0	-3	-3	-3	-6	-3	-1	-3	-4	-3	-7	-3	-2	-3	-5
k=4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4	-4	0	-4	-4
k=5	0	+3	+3	+3	+6	+3	+1	+3	+4	+3	+7	+3	+2	+3	+5
k=6	0	+2	+2	+2	+4	+2	+6	+2	0	+2	+2	+2	+4	+2	+6
k=7	0	+1	+1	+1	+2	+1	+3	+1	+4	+1	+5	+1	+6	+1	+7

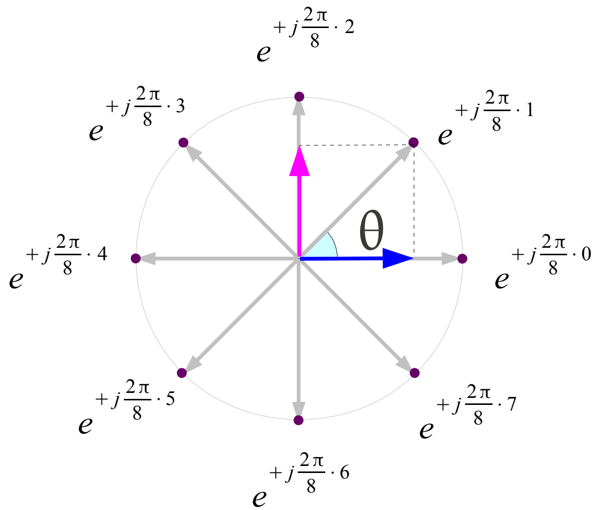


stride	angular speed		Measuring Frequency	Harmonic Frequency
0	cw	0	0	
-1	cw	-1ω	+1f	Fundamental → 1st harmonic
-2	cw	-2ω	+2f	2nd harmonic
-3	cw	-3ω	+3f	3rd harmonic
-4	cw	-4ω	+4f	4th harmonic
+3	ccw	+3ω	-3f	5th harmonic
+2	ccw	+2ω	-2f	6th harmonic
+1	ccw	+1ω	-1f	7th harmonic

Fundamental Frequency $$f_0 = \frac{1}{T}$$

Harmonic Frequency $$f_k = k \cdot f_0 \quad (k = 1,2,\cdots)$$

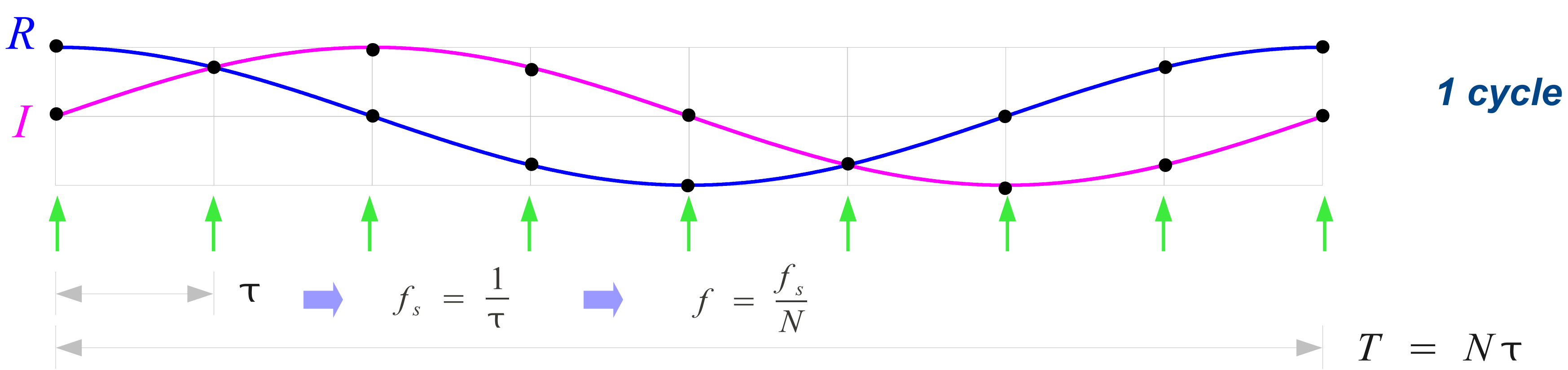
Sampling Frequency



Sampling Time	τ	
Sampling Frequency	$f_s = \dfrac{1}{\tau}$	*(samples per second)*

Period	$T = N\tau$	
Fundamental Freq	$f = \dfrac{1}{T}$	*(cycles per second)*
	$f = \dfrac{f_s}{N} \left(= \dfrac{1}{N\tau} \right)$	

Sinusoid with Fundamental Frequency



Normalized Frequency

Sampling Time τ

Sampling Frequency $f_s = \frac{1}{\tau}$ *(samples per second)*

τ

$T = N\tau$

Normalized Frequency

$$\frac{f_n}{f_s} = \frac{n}{N}$$ *(cycles per sample)*

Fundamental Frequency $f_0 = \frac{1}{T} = \frac{1}{N\tau}$ → $f_0 = \frac{f_s}{N}$

Harmonic Frequencies

$$f_1 = 1 \cdot f_0 = \frac{1}{N} f_s$$
$$f_2 = 2 \cdot f_0 = \frac{2}{N} f_s$$
$$f_3 = 3 \cdot f_0 = \frac{3}{N} f_s$$
$$\cdots$$
$$f_{N-1} = (N-1) \cdot f_0 = \frac{(N-1)}{N} f_s$$

Normalized Frequencies

$$\frac{1}{N}$$
$$\frac{2}{N}$$
$$\frac{3}{N}$$
$$\cdots$$
$$\frac{(N-1)}{N}$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann