

# Laurent Series and z-Transform Examples case 0.A

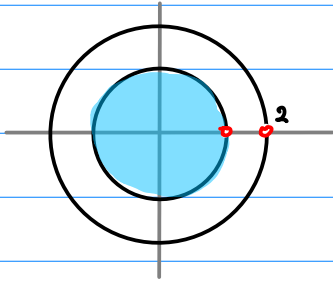
20171104

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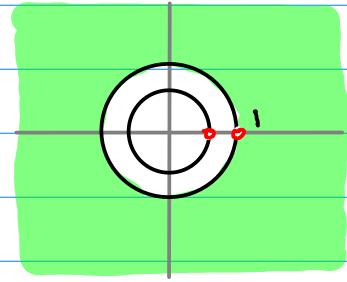
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1.A

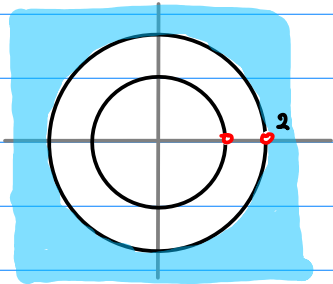
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$



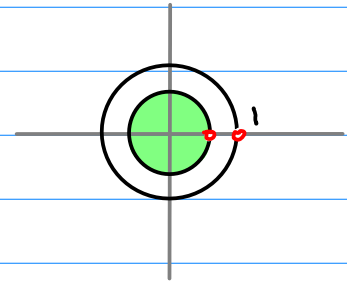
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



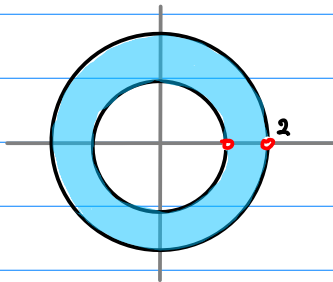
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



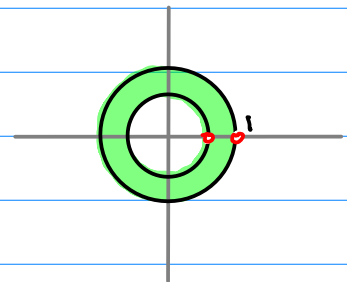
$$\sum_{n=-1}^{-\infty} \left( 1 - \left(\frac{1}{2}\right)^{n+1} \right) z^n$$



$$\sum_{n=-1}^{-\infty} \left( 1 - \left(\frac{1}{2}\right)^{n+1} \right) z^n$$



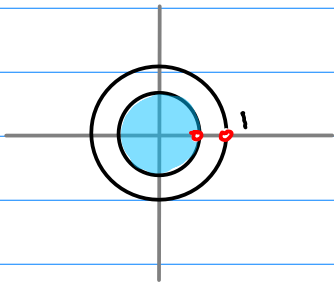
$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} \cdot z^n$$



$$\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} \cdot z^n$$

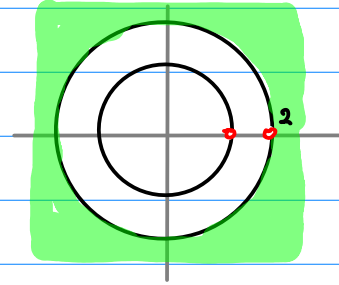
2.A

$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

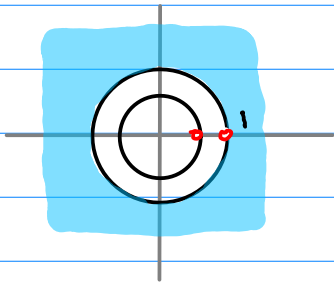


$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$

≡

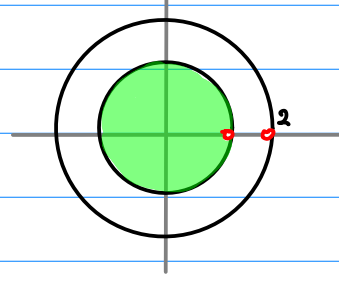


$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$

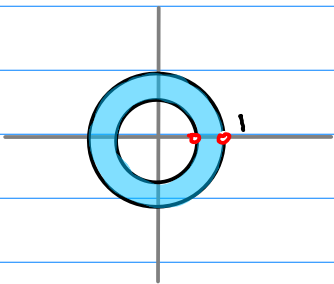


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

≡

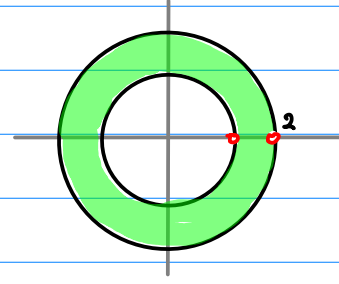


$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$



$$\sum_{n=-\infty}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

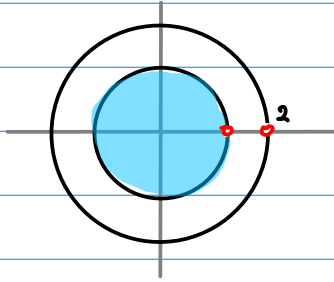
≡



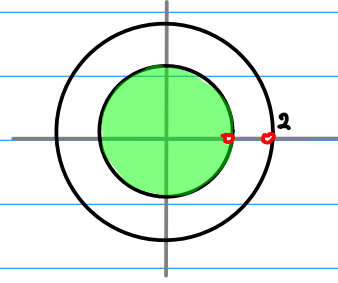
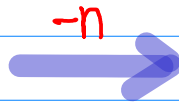
$$\sum_{n=-\infty}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

3. A

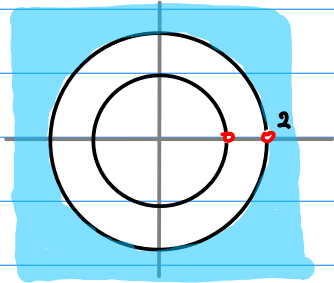
$$f(z) = \frac{-1}{(z-1)(z-2)} = X(z) = \frac{-1}{(z-1)(z-2)}$$



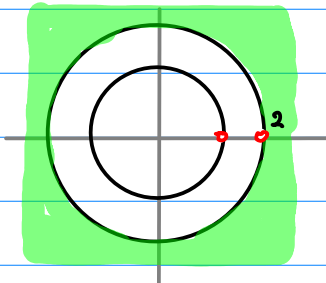
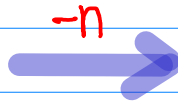
$$\sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



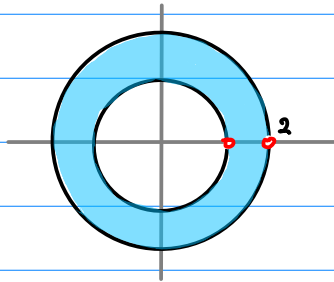
$$\sum_{n=0}^{-\infty} [2^{n-1} - 1] z^{-n}$$



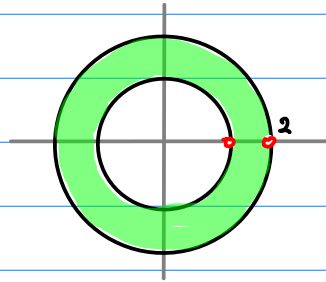
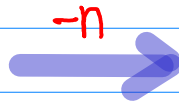
$$\sum_{n=-1}^{\infty} \left[ 1 - \left(\frac{1}{2}\right)^{n+1} \right] z^n$$



$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^{-n}$$



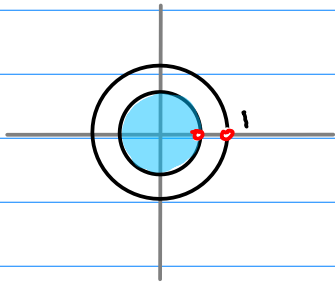
$$\sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n$$



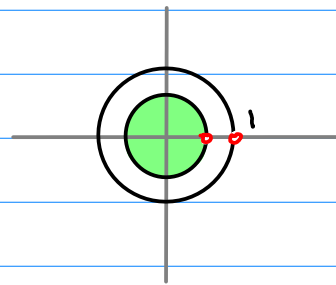
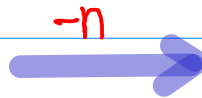
$$+ \sum_{n=-1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.A

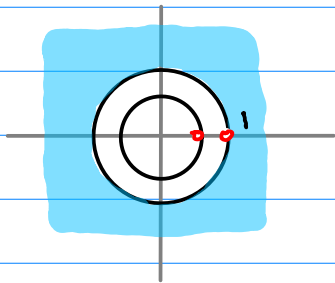
$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



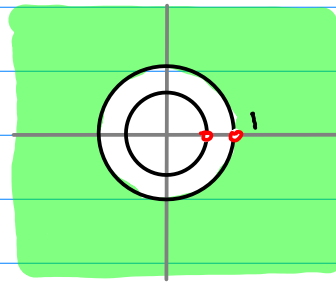
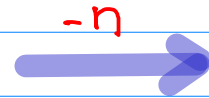
$$\sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$



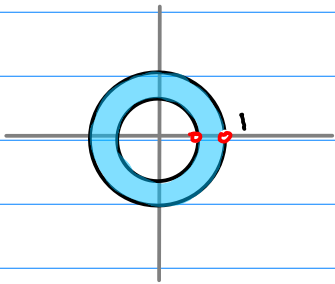
$$\sum_{n=-\infty}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$



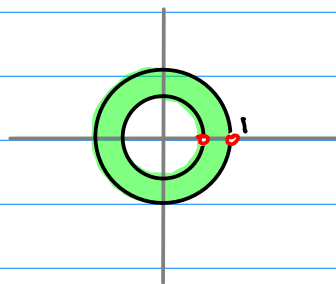
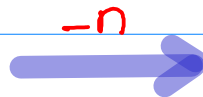
$$\sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^n$$



$$\sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$



$$+\sum_{n=-\infty}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n$$

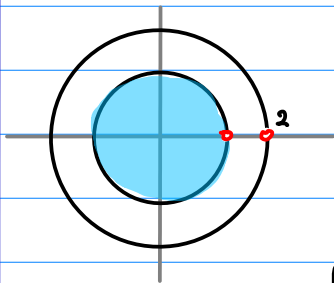


$$+\sum_{n=-\infty}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

1. A

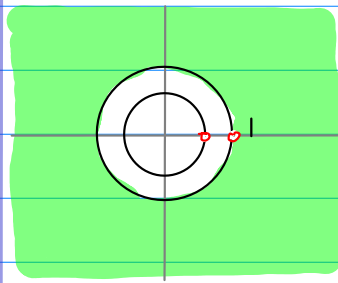
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

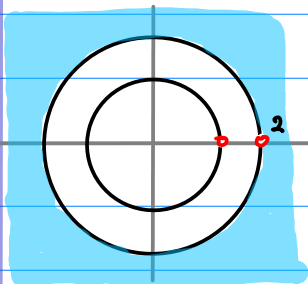
$$f(z) = \sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ (\frac{1}{2})^{n+1} - 1 & (n \leq 0) \end{cases}$$

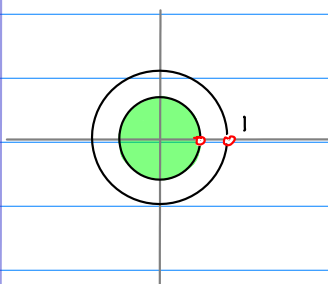
$$X(z) = \sum_{n=0}^{\infty} \left[ \left(\frac{1}{2}\right)^{n+1} - 1 \right] z^n$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ (1 - (\frac{1}{2})^{n+1}) & (n < 0) \end{cases}$$

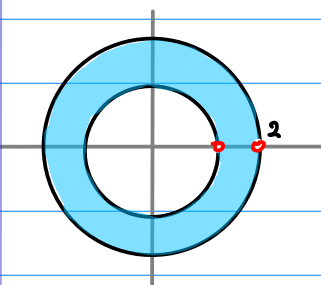
$$f(z) = \sum_{n=-1}^{-\infty} (1 - (\frac{1}{2})^{n+1}) z^n$$



$$x_n = \begin{cases} (1 - (\frac{1}{2})^{n+1}) & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

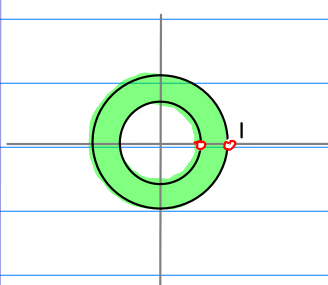
$$X(z) = \sum_{n=-1}^{-\infty} (1 - (\frac{1}{2})^{n+1}) z^n$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$



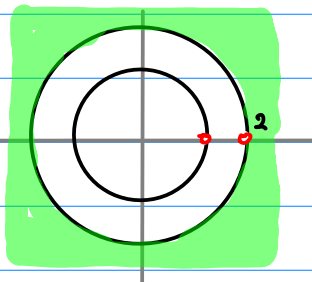
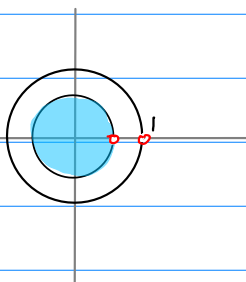
$$x_n = \begin{cases} 1 & (n > 0) \\ (\frac{1}{2})^{n+1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$

2.A

$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

I



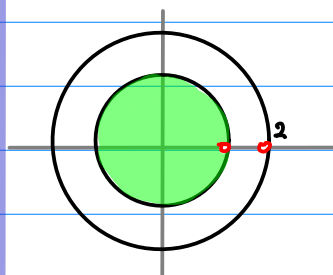
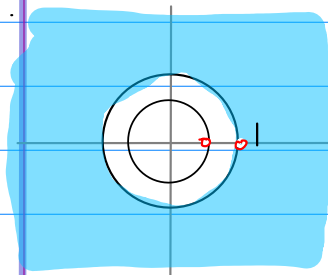
$$a_n = \begin{cases} [1 - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} 0 & (n \geq 0) \\ [1 - 2^{n-1}] & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n$$

$$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n$$

II



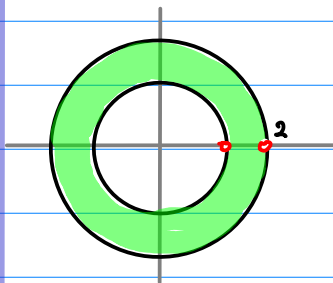
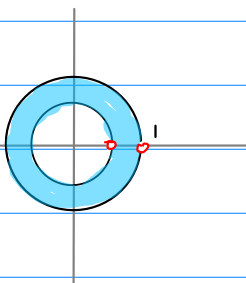
$$a_n = \begin{cases} 0 & (n > 0) \\ [2^{n-1} - 1] & (n \leq 0) \end{cases}$$

$$x_n = \begin{cases} [2^{n-1} - 1] & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

$$X(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1] z^n$$

III



$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

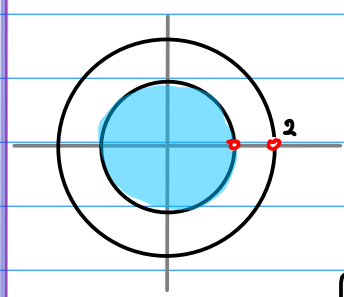
$$x_n = \begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^n$$

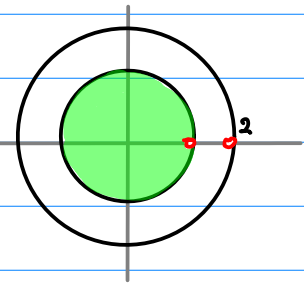
3.A  $f(z) = \frac{-1}{(z-1)(z-2)} = X(z) = \frac{-1}{(z-1)(z-2)}$

I



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

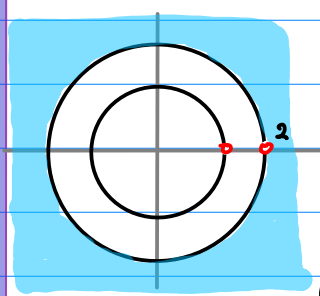
$$f(z) = \sum_{n=0}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^n$$



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

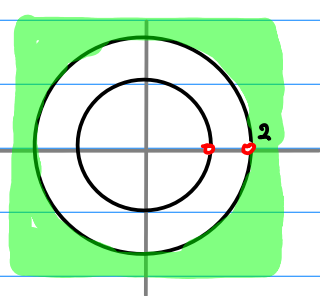
$$X(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

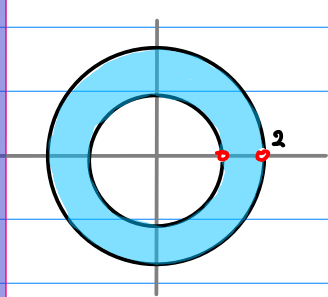
$$f(z) = \sum_{n=-1}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^n$$



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

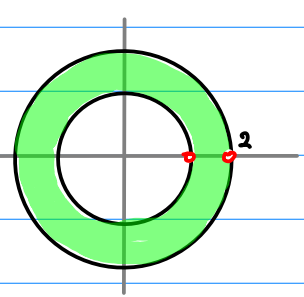
$$X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^{-n}$$

III



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{\infty} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$



$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

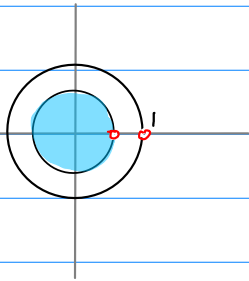
$$X(z) = \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$



4.A

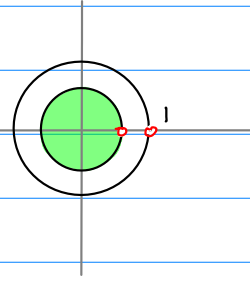
$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$

I



$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

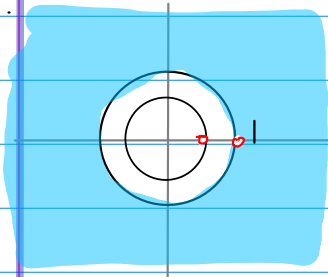
$$f(z) = \sum_{n=-\infty}^{\infty} [1 - 2^{n-1}] z^n$$



$$x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

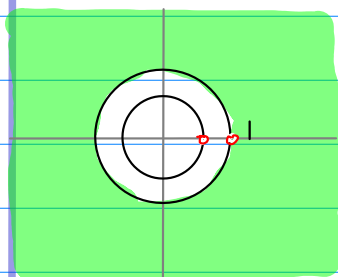
$$X(z) = \sum_{n=-\infty}^{\infty} [1 - (\frac{1}{2})^{n+1}] z^{-n}$$

II



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

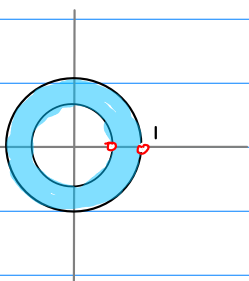
$$f(z) = \sum_{n=-\infty}^{\infty} [2^{n-1} - 1] z^n$$



$$x_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

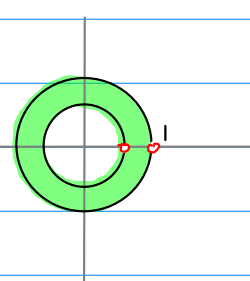
$$X(z) = \sum_{n=-\infty}^{\infty} [(\frac{1}{2})^{n+1} - 1] z^{-n}$$

III



$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$f(z) = + \sum_{n=-\infty}^{\infty} z^n + \sum_{n=0}^{\infty} 2^{n-1} z^n$$



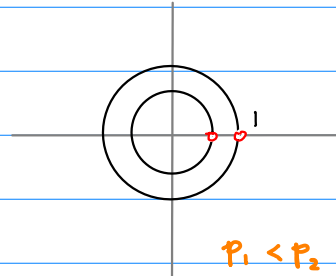
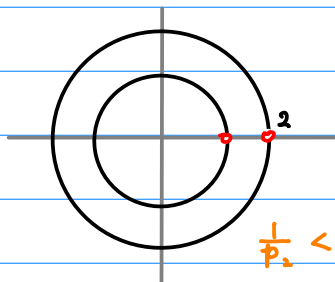
$$x_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$X(z) = + \sum_{n=-\infty}^{\infty} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n}$$

$$f(z) \xrightarrow{z^{-1}} X(z)$$

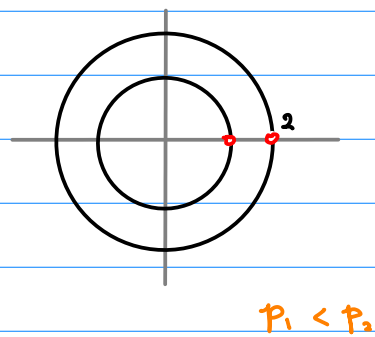
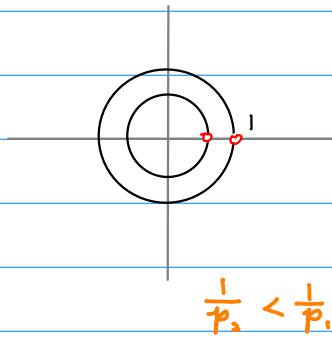
1.A

$$\frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} \frac{-0.5z^2}{(z-1)(z-0.5)}$$



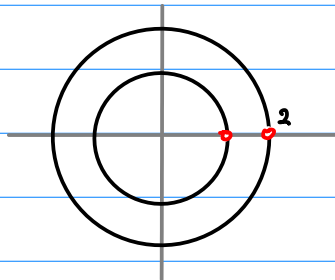
1.B

$$\frac{-0.5z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} \frac{-1}{(z-1)(z-2)}$$

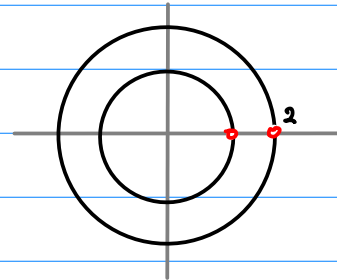


$$f(z) = X(z)$$

3.A  $\frac{-1}{(z-1)(z-2)} = \frac{-1}{(z-1)(z-2)}$

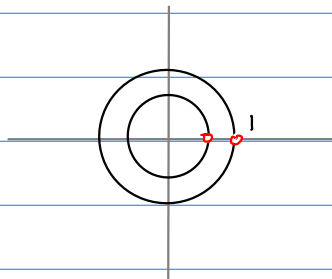


$$p_1 < p_2$$

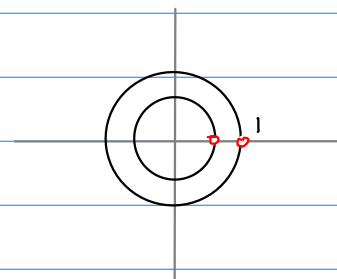


$$p_1 < p_2$$

4.B  $\frac{-0.5z^2}{(z-1)(z-0.5)} = \frac{-0.5z^2}{(z-1)(z-0.5)}$



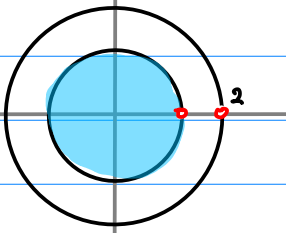
$$p_1 < p_2$$



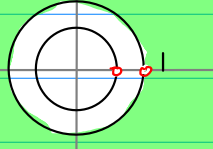
$$p_1 < p_2$$

$$f(z) \xrightarrow{z^{-1}} X(z)$$

**I**

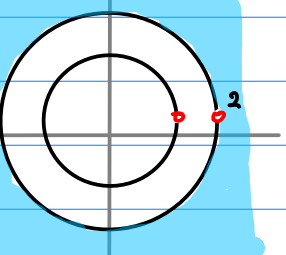


$(\frac{1}{2})^{n+1} - 1$	$(n \geq 0)$
0	$(n < 0)$

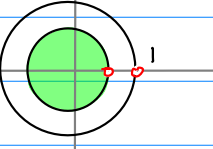


0	$(n > 0)$
$(\frac{1}{2})^{n+1} - 1$	$(n \leq 0)$

**II**

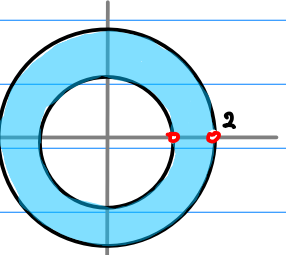


0	$(n \geq 0)$
$1 - (\frac{1}{2})^{n+1}$	$(n < 0)$

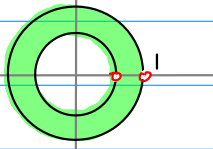


$1 - (\frac{1}{2})^{n+1}$	$(n > 0)$
0	$(n \leq 0)$

**III**

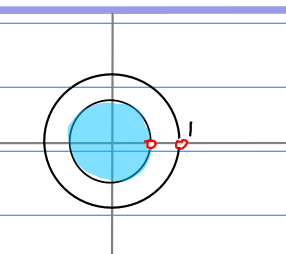


$(\frac{1}{2})^{n+1}$	$(n \geq 0)$
1	$(n < 0)$

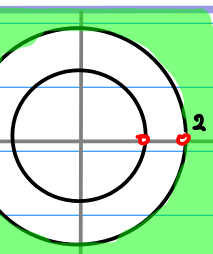


1	$(n > 0)$
$(\frac{1}{2})^{n+1}$	$(n \leq 0)$

**I**

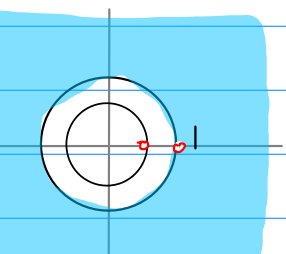


$1 - 2^{n-1}$	$(n > 0)$
0	$(n \leq 0)$

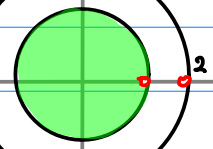


0	$(n \geq 0)$
$1 - 2^{n-1}$	$(n < 0)$

**II**

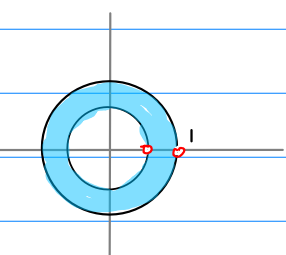


0	$(n > 0)$
$2^{n-1} - 1$	$(n \leq 0)$

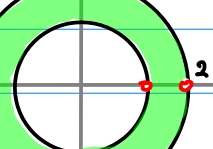


$2^{n-1} - 1$	$(n \geq 0)$
0	$(n < 0)$

**III**



1	$(n > 0)$
$2^{n-1}$	$(n \leq 0)$



$2^{n-1}$	$(n \geq 0)$
1	$(n < 0)$

$$f(z) = X(z)$$

I		$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$		$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$
II		$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}$		$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$
III		$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$		$\begin{matrix} 1 & (n > 0) \\ 2^n & (n \leq 0) \end{matrix}$
I		$\begin{matrix} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{matrix}$		$\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}$
II		$\begin{matrix} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{matrix}$		$\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}$
III		$\begin{matrix} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{matrix}$		$\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}$

$$f(z) \xrightarrow{z^{-1}} X(z)$$

I

$$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} 0 & (n > 0) \\ (\frac{1}{2})^{n+1} - 1 & (n \leq 0) \end{cases}$$

\* -1

II

$$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 1 - (\frac{1}{2})^{n+1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

III

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} 1 & (n > 0) \\ (\frac{1}{2})^{n+1} & (n \leq 0) \end{cases}$$

I

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - 2^{n-1} & (n < 0) \end{cases}$$

\* -1

II

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$\begin{cases} 2^{n-1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

III

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$\begin{cases} 2^{n-1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = X(z)$$

Ⓘ

$$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

\*-1

Ⓜ

$$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

\*-1

Ⓝ

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

Ⓘ

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

\*-1

Ⓜ

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$$\begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

\*-1

Ⓝ

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$\begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$f(z)$

$p_1 < p_2$

$|z| < p_1$

$\frac{1}{2}$

$\frac{1}{1}$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \quad (n \geq 0) \\ 0 \quad (n < 0) \end{array}$$

$p_1 = 1 \quad p_2 = 2$

\* -

$|z| > p_2$

$$\begin{array}{l} 0 \quad (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \quad (n < 0) \end{array}$$

$p_1 = 1 \quad p_2 = 2$

$p_1 < |z| < p_2$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \quad (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} \quad (n < 0) \end{array}$$

$p_1 = 1 \quad p_2 = 2$



$X(z)$  $p_1 < p_2$  $|z| < p_1$ 

$$\begin{array}{l} 0 \\ (p_1)^{n-1} - (p_2)^{n-1} \end{array} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

$p_1 = 1/2 \quad p_2 = 1$

 $*-1$  $|z| > p_2$ 

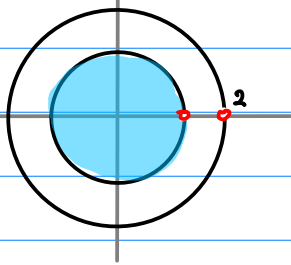
$$\begin{array}{l} (p_2)^{n-1} - (p_1)^{n-1} \\ 0 \end{array} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

$p_1 = 1/2 \quad p_2 = 1$

 $p_1 < |z| < p_2$ 

$$\begin{array}{l} (p_2)^{n-1} \\ (p_1)^{n-1} \end{array} \quad \begin{array}{l} (n > 0) \\ (n \leq 0) \end{array}$$

I

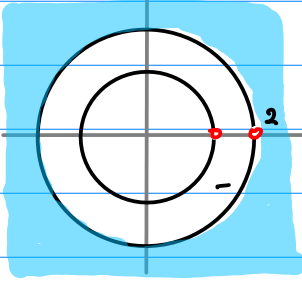


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

II

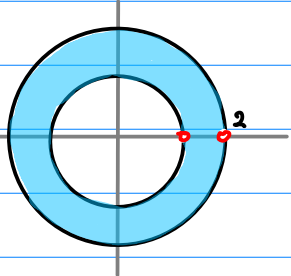


$$\begin{array}{ll} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

III

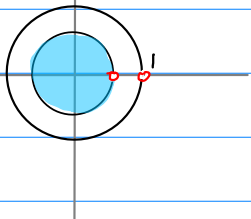


$$\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{array}$$

$$p_1 = 1 \quad p_2 = 2$$

I

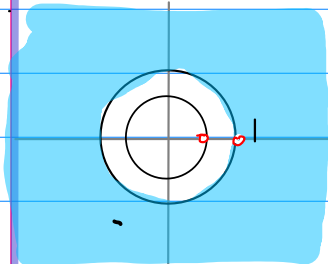


$$\begin{array}{ll} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{array}$$

$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1} \\ 0 \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

II

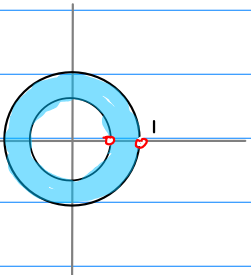


$$\begin{array}{ll} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{array}$$

$$\begin{array}{l} 0 \\ \left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1} \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

III



$$\begin{array}{ll} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{array}$$

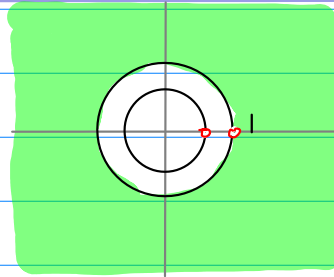
$$\begin{array}{l} \left(\frac{1}{p_2}\right)^{n-1} \\ \left(\frac{1}{p_1}\right)^{n-1} \end{array}$$

$$p_1 = \frac{1}{2} \quad p_2 = 1$$

I

$$\frac{0}{(p_1)^{n+1} - (p_2)^{n+1}}$$

$$p_1 = 1/2 \quad p_2 = 1$$

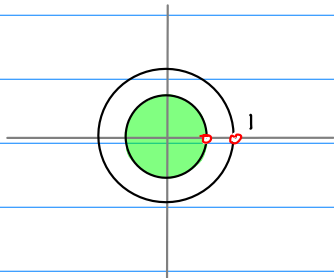


$$\begin{aligned} 0 & \quad (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} - 1 & \quad (n \leq 0) \end{aligned}$$

II

$$\frac{(p_2)^{n+1} - (p_1)^{n+1}}{0}$$

$$p_1 = 1/2 \quad p_2 = 1$$

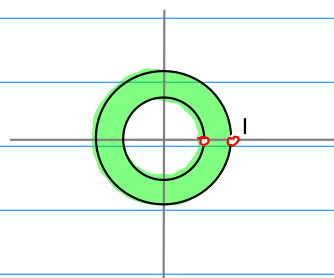


$$\begin{aligned} 1 - \left(\frac{1}{2}\right)^{n+1} & \quad (n > 0) \\ 0 & \quad (n \leq 0) \end{aligned}$$

III

$$\frac{(p_2)^{n+1}}{(p_1)^{n+1}}$$

$$p_1 = 1/2 \quad p_2 = 1$$

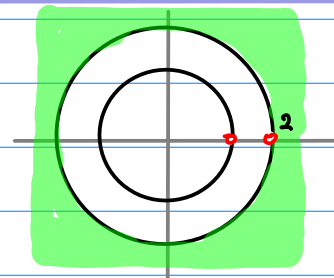


$$\begin{aligned} 1 & \quad (n > 0) \\ \left(\frac{1}{2}\right)^{n+1} & \quad (n \leq 0) \end{aligned}$$

I

$$\frac{0}{(p_1)^{n-1} - (p_2)^{n-1}}$$

$$p_1 = 1 \quad p_2 = 2$$

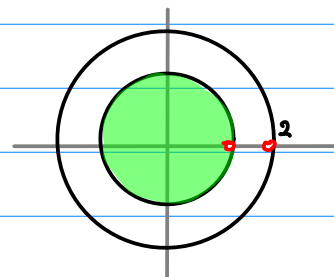


$$\begin{aligned} 0 & \quad (n \geq 0) \\ 1 - 2^{n-1} & \quad (n < 0) \end{aligned}$$

II

$$\frac{(p_2)^{n-1} - (p_1)^{n-1}}{0}$$

$$p_1 = 1 \quad p_2 = 2$$

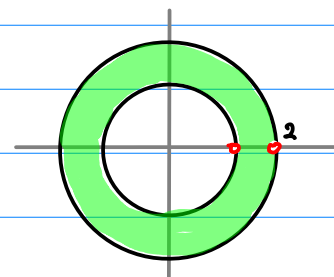


$$\begin{aligned} 2^{n-1} - 1 & \quad (n \geq 0) \\ 0 & \quad (n < 0) \end{aligned}$$

III

$$\frac{(p_2)^{n-1}}{(p_1)^{n-1}}$$

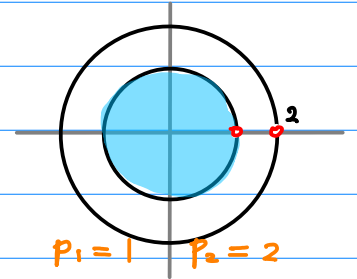
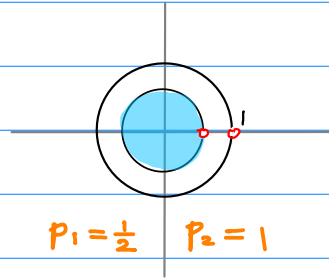
$$p_1 = 1 \quad p_2 = 2$$



$$\begin{aligned} 2^{n-1} & \quad (n \geq 0) \\ 1 & \quad (n < 0) \end{aligned}$$

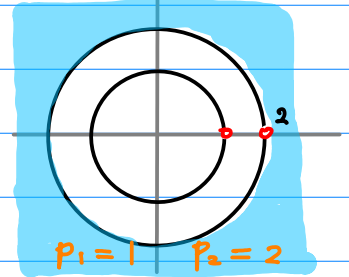
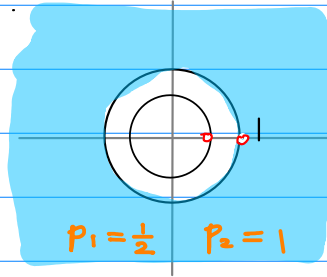
$$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$$

$$0$$



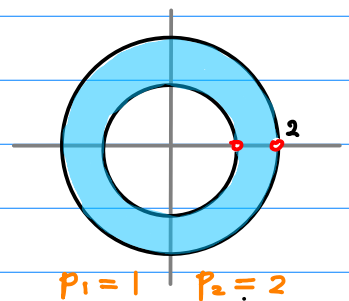
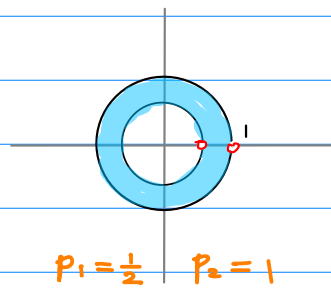
$$0$$

$$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$$



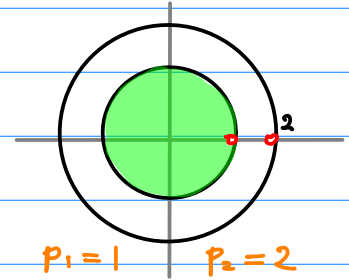
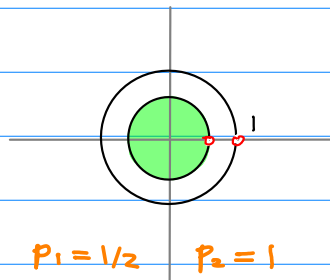
$$\left(\frac{1}{p_2}\right)^{n+1}$$

$$\left(\frac{1}{p_1}\right)^{n+1}$$



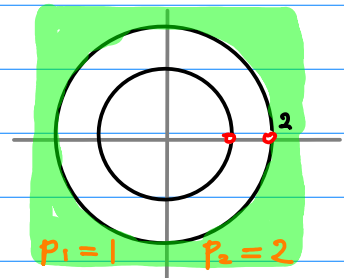
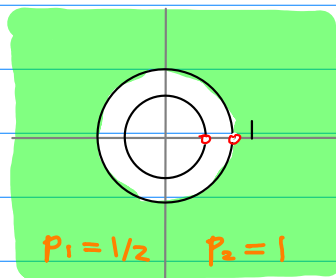
$$(p_2)^{n+1} - (p_1)^{n+1}$$

$$0$$



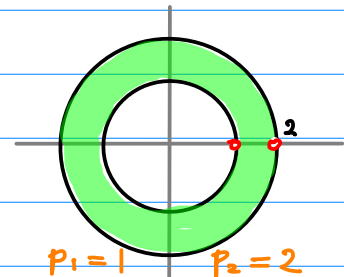
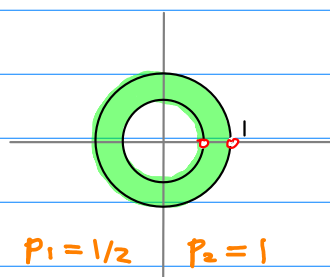
$$0$$

$$(p_1)^{n+1} - (p_2)^{n+1}$$

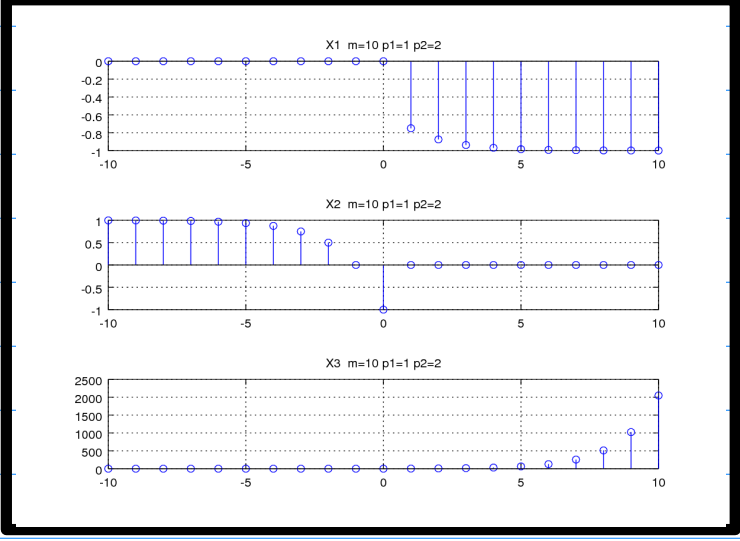
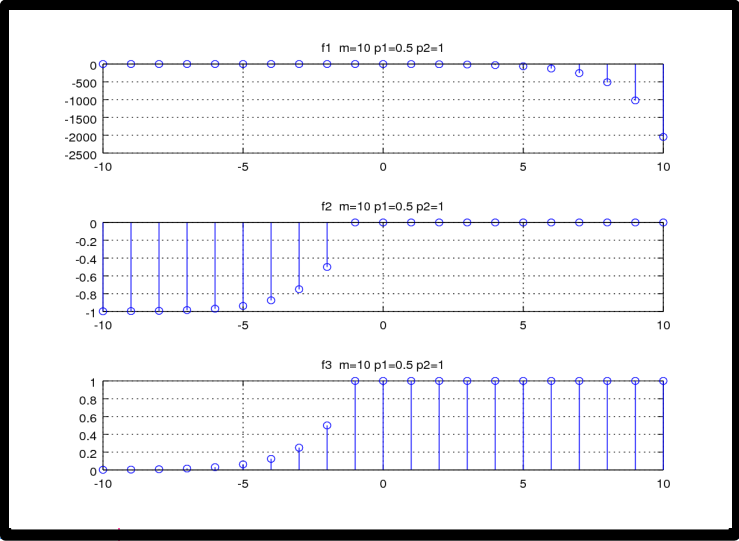
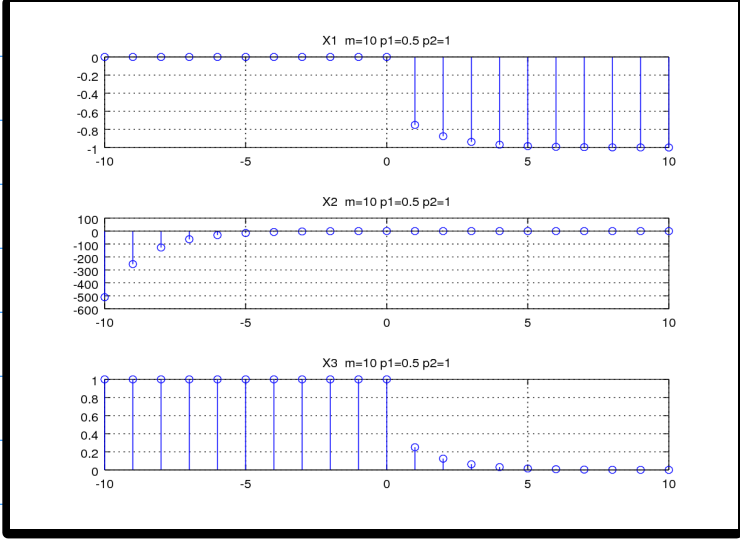
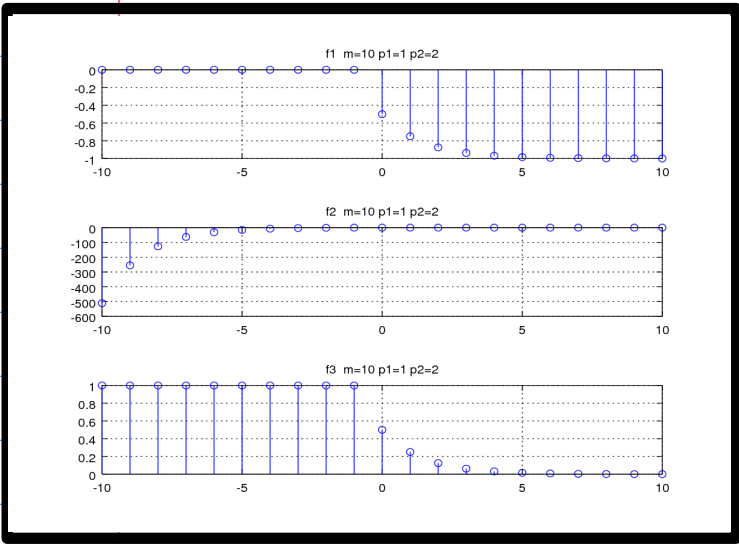


$$(p_2)^{n+1}$$

$$(p_1)^{n+1}$$







$P_1$   
 $P_2$   
 $m+1$   
 $p-1$   
 check.

```
% Laurent Series and sequences
```

```
function plotseq1(m=1, p1=2, p2=2.1)
```

```
t1p = 0 : m;
```

```
t1n = -m: -1;
```

```
t1 = [t1n, t1p];
```

```
f1 = [zeros(1,m), ((1/p2).^(t1p+1) - (1/p1).^(t1p+1))];
```

```
f2 = [((1/p1).^(t1n+1) - (1/p2).^(t1n+1)), zeros(1,m+1)];
```

```
f3 = [(1/p1).^(t1n+1), (1/p2).^(t1p+1)];
```

```
subplot(3, 1, 1);
```

```
stem(t1, f1);
```

```
grid on
```

```
%axis([0, m])
```

```
title(sprintf("f1 m=%d p1=%g p2=%g", m, p1, p2))
```

```
subplot(3, 1, 2);
```

```
stem(t1, f2);
```

```
grid on
```

```
%axis([0, m])
```

```
title(sprintf("f2 m=%d p1=%g p2=%g", m, p1, p2))
```

```
subplot(3, 1, 3);
```

```
stem(t1, f3);
```

```
grid on
```

```
%axis([0, m])
```

```
title(sprintf("f3 m=%d p1=%g p2=%g", m, p1, p2))
```

```
endfunction
```

```

% z-Transform and sequences
function plotseq2(m=1, p1=2, p2=2.1)

cla;

t1p = 1 : m;
t1n = -m: 0;
t1 = [t1n, t1p];
X1 = [zeros(1,m+1), ((p1).^(t1p-1) - (p2).^(t1p-1))];
X2 = [((p2).^(t1n-1) -(p1).^(t1n-1)), zeros(1,m)];
X3 = [(p2).^(t1n-1), (p1).^(t1p-1)];

subplot(3, 1, 1);
stem(t1, X1);
grid on
%axis([0, m])
title(sprintf("X1 m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, X2);
grid on
%axis([0, m])
title(sprintf("X2 m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, X3);
grid on
%axis([0, m])
title(sprintf("X3 m=%d p1=%g p2=%g", m, p1, p2))

endfunction

```